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Keywords Newton's Law · teleparallelism · R^2-gravity · MOND

1 Introduction

Being a 'close relative' of General Relativity (GR), Absolute Parallelism (AP) has many interesting features: larger symmetry group of equations; field irreducibility with respect to this group; vast list of compatible second order equations (discovered by Einstein and Mayer [1]) not restricted to Lagrangian ones.

There is the only variant of Absolute Parallelism which solutions are free of arising singularities, if D=5 (there is no room for changes; this variant of AP does not have a Lagrangian, nor match GR); in this case AP has topological features of nonlinear sigma-model.

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In order to give clear presentation and full picture of the theory' scope, many items should be sketched: instability of trivial solution and expanding O_4-symmetrical ones; tensor $T_{\mu\nu}$ (positive energy, but only three polarizations of 15 carry (and angular) momentum; how to quantize such a stuff ?) and PN-effects; topological classification of symmetric 5D field configurations (alighting on evident parallels with Standard Model' particle combinatorics) and 'quantum phenomenology on expanding classical background' (coexistence); 'plain' R^2-gravity on very thick brane and change in the Newton's Law: $\frac{1}{r^2}$ goes to $\frac{1}{r}$ with distance (not with acceleration – as it is in MOND [2]).

At last, an experiment with single photon interference is discussed as the other way to observe very-very long (and very undeveloped) the extra dimension.

2 Unique 5D equation of AP (free of singularities in solutions)

There is one unique variant of AP (non-Lagrangian, with the unique D; D=5) which solutions of general position seem to be free of arising singularities. The formal integrability test [3] can be extended to the cases of degeneration of either co-frame matrix, $h^a{}_\mu$, (co-singularities) or contra-variant frame (or contra-frame density of some weight), serving as the local and covariant (no coordinate choice) test for singularities of solutions. In AP this test singles out the next equation (and D=5, see [4]; $\eta_{ab} = \mathrm{diag}(-1,1,\ldots,1)$, then $h = \det h^a{}_\mu = \sqrt{-g}$):

$$\mathbf{E}_{a\mu} = L_{a\mu\nu;\nu} - \tfrac{1}{3}(f_{a\mu} + L_{a\mu\nu}\Phi_\nu) = 0\,, \tag{1}$$

where (see [4] for more detailed introduction to AP and explanation of notations used)

$$L_{a\mu\nu} = L_{a[\mu\nu]} = \Lambda_{a\mu\nu} - S_{a\mu\nu} - \tfrac{2}{3}h_{a[\mu}\Phi_{\nu]}\,,$$
$$\Lambda_{a\mu\nu} = 2h_{a[\mu,\nu]}\,,\ S_{\mu\nu\lambda} = 3\Lambda_{[\mu\nu\lambda]}\,,\ \Phi_\mu = \Lambda_{aa\mu}\,,\ f_{\mu\nu} = 2\Phi_{[\mu,\nu]} = 2\Phi_{[\mu;\nu]}\,. \tag{2}$$

Coma "," and semicolon ";" denote partial derivative and usual covariant differentiation with symmetric Levi-Civita connection, respectively.

One should retain the identities [which follow from the definitions (2)]:

$$\Lambda_{a[\mu\nu;\lambda]} \equiv 0\,,\ \ h_{a\lambda}\Lambda_{abc;\lambda} \equiv f_{cb}\,(= f_{\mu\nu}h^\mu_c h^\nu_b)\,,\ \ f_{[\mu\nu;\lambda]} \equiv 0. \tag{3}$$

The equation $\mathbf{E}_{a\mu;\mu} = 0$ gives 'Maxwell-like equation' (we prefer to omit $g^{\mu\nu}$ (η^{ab}) in contractions that not to keep redundant information – when covariant differentiation is in use only):

$$(f_{a\mu} + L_{a\mu\nu}\Phi_\nu)_{;\mu} = 0,\ \text{or}\ \ f_{\mu\nu;\nu} = (S_{\mu\nu\lambda}\Phi_\lambda)_{;\nu}\ \ (= -\tfrac{1}{2}S_{\mu\nu\lambda}f_{\nu\lambda})\,. \tag{4}$$

Actually the Eq. (4) follows from the symmetric part of equation, $\mathbf{E}_{(ab)}$, because skewsymmetric one gives just the identity:

$$2\mathbf{E}_{[\nu\mu]} = S_{\mu\nu\lambda;\lambda} = 0,\ \mathbf{E}_{[\mu\nu];\nu} \equiv 0;$$

note also that the trace part becomes irregular (the principal derivatives vanish) if $D = 4$ (this number of dimension is forbidden, and the next number, $D = 5$, is the most preferable):

$$\mathbf{E}_{\mu\mu} = \mathbf{E}_{a\mu}h^\mu_b\eta^{ab} = \tfrac{4-D}{3}\Phi_{\mu;\mu} + (\Lambda^2) = 0.$$

The system (1) remains compatible under adding $f_{\mu\nu}=0$, see (4); this is not the case for another covariant, S, Φ, or (some irreducible part of the) Riemannian curvature, which relates to Λ as usually:

$$R_{a\mu\nu\lambda} = 2h_{a\mu;[\nu;\lambda]};\ h_{a\mu}h_{a\nu;\lambda} = \tfrac{1}{2}S_{\mu\nu\lambda} - \Lambda_{\lambda\mu\nu}.$$

3 Tensor $T_{\mu\nu}$ (despite Lagrangian absence) and PN-effects

One might rearrange $\mathbf{E}_{(\mu\nu)}=0$ that to pick out (into LHS) the Einstein tensor, $G_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R$, but the rest terms are not proper energy-momentum tensor: they contain linear terms $\Phi_{(\mu;\nu)}$ (no positive energy (!); another presentation of 'Maxwell equation' (4) is possible instead – as divergence of symmetrical tensor).

However, the prolonged equation $\mathbf{E}_{(\mu\nu);\lambda;\lambda}=0$ can be written as 'plain' (no R-term) R^2-gravity:

$$G_{\mu\nu;\lambda;\lambda} + G_{\varepsilon\tau}(2R_{\varepsilon\mu\tau\nu} - \tfrac{1}{2}g_{\mu\nu}R_{\varepsilon\tau}) = T_{\mu\nu}(\Lambda'^2,\ldots) \qquad (5)$$

(RHS is just $-h^{-1}\,\delta(hR_{\mu\nu}G^{\mu\nu})/\delta g_{\mu\nu}$, so $T_{\mu\nu;\nu}=0$); up to quadratic terms,

$$T_{\mu\nu} = \tfrac{2}{9}(\tfrac{1}{4}g_{\mu\nu}f^2 - f_{\mu\lambda}f_{\nu\lambda}) + A_{\mu\varepsilon\nu\tau}(\Lambda^2)_{;(\varepsilon;\tau)} + (\Lambda^2\Lambda', \Lambda^4);$$

tensor A has symmetries of Riemann tensor, so the term A'' adds nothing to momentum and angular momentum.

It is worth noting that:
(a) the theory does not match GR, but shows 'plain' R^2-gravity (sure, (5) does not contain all the theory);
(b) only f-component (three transverse polarizations in D=5) carries D-momentum and angular momentum ('powerful' waves); other 12 polarizations are 'powerless', or 'weightless' (this is a very unusual feature – impossible in the Lagrangian tradition; how to quantize ? let us not to try this, leaving the theory 'as is');
(c) f-component feels only metric and S-field ('contorsion', not 'torsion' Λ – to label somehow), see (4), but S has effect only on polarization of f: $S_{[\mu\nu\lambda]}$ does not enter eikonal equation, and f moves along usual Riemannian geodesic (if background has f=0); one may think that all 'quantum fields' (phenomenological quantized fields accounting for topological (quasi)charges and carrying some 'power'; see further) inherit this property;
(d) the trace $T_{\mu\mu}=\frac{1}{18}f_{\mu\nu}f_{\mu\nu}$ can be non-zero if $f^2\neq 0$ and this seemingly depends on S-component [which enters the current in (4)]; in other words, 'mass distribution' is to depend on distribution of f- and S-component;
(e) it should be stressed and underlined that the f-component is not usual (quantum) EM-field – just important covariant responsible for energy-momentum (suffice it to say that there is no gradient invariance for f).

4 Linear domain: instability of trivial solution (with powerless waves

Another strange feature is the instability of trivial solution: some 'powerless' polarizations grow linearly with time in presence of 'powerful' f-polarizations. Really, from the linearized Eq. (1) and the identity (3) one can write (the following equations should be understood as linearized; $D \neq 4$):

$$\Phi_{a,a} = 0,\ 3\Lambda_{abd,d} = \Phi_{a,b} - 2\Phi_{b,a},\ \Lambda_{a[bc,d],d} \equiv 0\ \Rightarrow\ 3\Lambda_{abc,dd} = -2 f_{bc,a}\,.$$

The last ‘D‘Alembert equation’ has the ‘source’ in its right hand side. Some components of Λ (most symmetrical irreducible parts) do not grow (as well as curvature), because (again, linearized equations are implied)

$$S_{abc,dd} = 0,\ \Phi_{a,dd} = 0,\ \ f_{ab,dd} = 0,\ R_{abcd,ee} = 0,$$

but the least symmetrical components of the tensor Λ do grow up with time (due to terms $\sim t\, e^{-i\omega t}$; three growing polarizations which are ‘imponderable’, or powerless) if the ‘ponderable’ waves (three f-polarizations) do not vanish (and this should be the case for solutions of ‘general position’).

5 Expanding O$_4$-symmetrical (single wave) solutions and cosmology

The unique symmetry of AP equations gives scope for symmetrical solutions. In contrast to GR, this variant of AP has non-stationary spherically (O_4-) symmetric solutions. The O_4-symmetric frame field can be generally written as follows [4]:

$$h^a{}_\mu(t,x^i) = \begin{pmatrix} a & bn_i \\ cn_i & en_in_j + d\Delta_{ij} \end{pmatrix};\ \ i,j = (1,2,3,4),\ n_i = \frac{x^i}{r}. \qquad (6)$$

Here $a,\dots,e$ are functions of time, $t = x^0$, and radius r, $\Delta_{ij} = \delta_{ij} - n_in_j$, $r^2 = x^ix^i$. As functions of radius, b,c are odd, while the others are even; other boundary conditions: $e = d$ at $r = 0$, and $h^a{}_\mu \rightarrow \delta^a_\mu$ as $r \rightarrow \infty$. Placing in (6) $b = 0, e = d$ (the other interesting choice is b=c=0) and making integrations one can arrive to the next system (resembling dynamics of Chaplygin gas; dot and prime denote derivation on time and radius, resp.; $A = a/e = e^{1/2}$, $B = -c/e$):

$$A^{\cdot} = AB' - BA' + 3AB/r\ ,\ \ B^{\cdot} = AA' - BB' - 2B^2/r\ . \qquad (7)$$

This system (does not suffer of gradient catastrophe and) has non-stationary solutions; a single-wave solution of proper ‘amplitude’ might serve as a suitable cosmological (expanding) background.

The condition $f_{\mu\nu}$=0 is a must for solutions with such a high symmetry (as well as $S_{\mu\nu\lambda}$=0); so, these O_4-solutions carry no energy, that is, weight nothing (some lack of *gravity* ! in this theory the universe expansion seemingly has little common with *gravity*, GR and its dark energy [5]).

More realistic cosmological model might look like a single O_4-wave (or a sequence of such waves) moving along the radius and being filled with chaos, or stochastic waves, both powerful (*weak*, $\Delta h \ll 1$) and powerless ($\Delta h < 1$, but intense enough that to lead to non-linear fluctuations with $\Delta h \sim 1$), which form statistical ensemble(s) having a few characteristic parameters (after ‘thermalization’). The development and examination of stability of such a model is an interesting problem, but at the moment some speculations are allowable.

The metric variation in cosmological O_4-wave can serve as a time-dependent ‘shallow dielectric guide’ for that weak noise waves. The ponderable waves (which slightly 'decelerate' the O_4-wave) should have wave-vectors almost tangent to the S^3-sphere of wave-front that to be trapped inside this ('shallow') wave-guide; the imponderable waves can grow up, and partly escape from the wave-guide, and their wave-vectors can be less tangent to the S^3-sphere.

The waveguide thickness can be small for an observer in the center of O_4-symmetry, but in co-moving coordinates it can be very large (due to relativistic effect), however still small with respect to the radius of sphere, $L \ll R$. It seems that the radial dimension has to be very 'undeveloped'; that is, there are no other characteristic scales, smaller than L, along this extra-dimension.

6 Non-linear domain: topological charges and quasi-charges

Let AP-space is of trivial topology: no worm-holes, no compactified space dimensions, no singularities. One can continuously deform frame field $h(x)$ to a field of rotation matrices (metric can be diagonalized and 'square-rooted') $h^a{}_\mu(x) \to s^a{}_\mu(x) \in SO(1,d)$; $m{=}D{-}1$. Further deformation can remove boosts too, and so, for any space-like (Cauchy) surface, this gives a (pointed) map,

$$s: \mathbf{R}^m \cup \infty = S^m \to SO_m;\ \infty \mapsto 1^m \in SO_m.$$

The set of such maps consists of homotopy classes forming the group of topological charge, $\Pi(m)$:

$$\Pi(m) = \pi_m(SO_m);\ \ \Pi(3) = Z,\ \Pi(4) = Z_2 + Z_2. \tag{8}$$

Here Z is the infinite cyclic group, and Z_2 is the cyclic group of order two.

It is important that deformation to s-field can keep symmetry of field configuration. Definition: localized field (pointed map) $s(x): \mathbf{R}^m \to SO(m),\ s(\infty) = 1^m$, is G-symmetric if, in some coordinates,

$$s(\sigma x) = \sigma s(x) \sigma^{-1}\ \ \forall\ \sigma \in G \subset O(m)\,. \tag{9}$$

The set of such fields $\mathscr{C}_G^{(m)}$ generally consists of separate, disconnected components – homotopy classes forming the 'topological quasi-charge group' denoted here as $\Pi(G;m) \equiv \pi_0(\mathscr{C}_G^{(m)})$. These QC-groups classify symmetrical localized configurations of frame field. Since field equation does not break symmetry, quasi-charge conserves; if symmetry is not exact (because of distant regions), quasi-charge is not exactly conserving value, and quasi-particle (of zero topological charge) can annihilate (or be created) during colliding with another quasi-particle.

There is the other problem. Let $G1 \supset G2$, such that there is a mapping (embedding) $i:\ \mathscr{C}_{G1}^{(m)} \to \mathscr{C}_{G2}^{(m)}$, which induces the homomorphism of QC-groups: $i_*: \Pi(G1;m) \to \Pi(G2;m)$, so one has to describe this morphism. Let us consider the simple (discreet) symmetry group P_1 with a plane of reflection symmetry:

$$P_1 = \{1, p_{(1)}\},\ \text{ where } p_{(1)} = \mathrm{diag}(-1,1,\ldots,1) = p_{(1)}^{-1}.$$

It is necessary to set field $s(x)$ on the half-space $\frac{1}{2}\mathbf{R}^m = \{x^1 \geq 0\}$, with additional condition imposed on the surface $\mathbf{R}^{m-1} = \{x_1 = 0\}$ (stationary points of P_1 group) where s has to commute with the symmetry [see (9)]:

$$p_{(1)}x = x\ \Rightarrow\ s(x) = p_{(1)} s p_{(1)}\ \Rightarrow\ s \in 1 \times SO_{m-1}.$$

Hence, accounting for the localization requirement, we have a diad map (relative spheroid; here D^m is an m-ball and S^{m-1} its surface) $(D^m; S^{m-1}) \to (SO_m; SO_{m-1})$, and topological classification of such maps leads to the relative (or diad) homotopy group ([6]; the last equality below follows due to fibration $SO_m/SO_{m-1} = S^{m-1}$):

$$\Pi(P_1; m) = \pi_m(SO_m; SO_{m-1}) = \pi_m(S^{m-1}).$$

Similar considerations (of group orbits and stationary points) lead to the following result:

$$\Pi(O_l; m) = \pi_{m-l+1}(SO_{m-l+1}; SO_{m-l}) = \pi_{m-l+1}(S^{m-l}).$$

If $l > 3$, there is the equality: $\Pi(SO_l; m) = \Pi(O_l; m)$, while for $l = 2, 3$ one can find:

$$\Pi(SO_3; m) = \pi_{m-2}(SO_2 \times SO_{m-2}; SO_{m-3}) = \pi_{m-2}(S^1 \times S^{m-3}),$$

$$\Pi(SO_2; m) = \pi_{m-1}(SO_m; SO_{m-2} \times SO_2) = \pi_{m-1}(RG_+(m, 2)).$$

The set of quaternions with absolute value one, $\mathbf{H}_1 = \{\mathbf{f},\ |\mathbf{f}| = 1\}$, forms a group under quaternion multiplication, $\mathbf{H}_1 \cong SU_2 = S^3$, and any $s \in SO_4$ can be represented as a pair of such quaternions [6], $(\mathbf{f}, \mathbf{g}) \in S^3_{(l)} \times S^3_{(r)},\ |\mathbf{f}| = |\mathbf{g}| = 1$:

$$x^* = sx \ \Leftrightarrow\ \mathbf{x}^* = \mathbf{f}\,\mathbf{x}\,\mathbf{g}^{-1} = \mathbf{f}\,\mathbf{x}\,\overline{\mathbf{g}};\ \ |\mathbf{x}| = |\mathbf{x}^*|.$$

The pairs $(\mathbf{f}, \mathbf{g})$ and $(–\mathbf{f}, –\mathbf{g})$ correspond to the same rotation s, that is,

$$SO_4 = S^3_{(l)} \times S^3_{(r)} / \pm\,.$$

Note that the symmetry condition (9) also splits into two parts:

$$\mathbf{f}(\mathbf{axb}^{-1}) = \mathbf{af}(\mathbf{x})\mathbf{a}^{-1},\ \mathbf{g}(\mathbf{axb}^{-1}) = \mathbf{bg}(\mathbf{x})\mathbf{b}^{-1}\ \forall(\mathbf{a}, \mathbf{b}) \in G \subset SO_4. \tag{10}$$

7 An example of SO_2-symmetric quaternion field

Let's consider an example of $SO_2\{2, 3\}-$symmetric $\mathbf{f}$–field configuration ($\mathbf{g}$=1), which carries both charge and SO_2-quasi-charge (left, of course), $\mathbf{f}(\mathbf{x})$: $\mathbf{H} = \mathbf{R}^4 \to \mathbf{H}_1$; $\mathbf{f}(\infty) = 1$. The symmetry condition (10) reads

$$\mathbf{f}(e^{\mathbf{i}\phi/2}\mathbf{x}e^{-\mathbf{i}\phi/2}) = e^{\mathbf{i}\phi/2}\mathbf{f}(\mathbf{x})e^{-\mathbf{i}\phi/2}. \tag{11}$$

We'll switch to 'double-axial' coordinates: $\mathbf{x} = ae^{\mathbf{i}\varphi} + be^{\mathbf{i}\psi}\mathbf{j}$. Let us use imaginary quaternions $\mathbf{q}$ as stereogrphic coordinates on $\mathbf{H}_1$, and take symmetrical field $\mathbf{q}(\mathbf{x})$ consistent with Eq. (11):

$$\mathbf{q}(\mathbf{x}) = \mathbf{x}\,\mathbf{i}\,\overline{\mathbf{x}} + \mathbf{i} = -\overline{\mathbf{q}},\ \mathbf{f}(\mathbf{x}) = -\frac{1+\mathbf{q}}{1-\mathbf{q}} = 1 - \frac{2}{1-\mathbf{q}}. \tag{12}$$

It is easy to find the 'center of quasi-soliton' (1-submanifold, S^1)

$$S^1 = \mathbf{f}^{-1}(-1) = \mathbf{q}^{-1}(0) = \{a = 0,\ b = 1\} = \{\mathbf{x}_0(\psi) = e^{\mathbf{i}\psi}\mathbf{j}\}$$

Table 1 QC-groups $\Pi_{(l)}(G)$ and their morphisms to the preceding group; $G \subset G_0$

G	$\Pi_l(G) \to \Pi_l(G^*)$	'label'
1	Z_2	
$SO\{1,2\}$	$Z_{(e)} \stackrel{e}{\to} Z_2$	e
$SO\{1,2\} \times P\{3,4\}$	$Z_{(\nu)} + Z_{(H)} \stackrel{i,m2}{\to} Z_{(e)}$	ν^0; $H^0 \to e + e$
$SO\{1,2\} \times P\{2,3\}$	$Z_{(W)} \stackrel{0}{\to} Z_{(e)}$	$W \to e + \nu^0$
$SO\{1,2\} \times P\{2,4\}$	$Z_{(Z)} \stackrel{0}{\to} Z_{(e)}$	$Z^0 \to e + e$
$SO\{1,2\} \times P\{3,4\} \times$ $\times P\{2,3\}$	$Z_{(\gamma)} \stackrel{0}{\to} Z_{(H)}$ $\stackrel{0}{\to} Z_{(W)}$	$\gamma^0 \to H^0 + H^0$ $\to W + W$

and the 'vector equipment' on this circle:

$$d\mathbf{x}|_{\mathbf{x}_0} = da\, e^{\mathbf{i}\varphi} + (db + \mathbf{i}\, d\psi) e^{\mathbf{i}\psi}\mathbf{j},\ \tfrac{1}{4} d\mathbf{f}\Big|_{\mathbf{x}_0} = \mathbf{i} db - \mathbf{k}\, e^{\mathbf{i}(\varphi+\psi)} da;$$

i-vector all time looks along the radius b (parallel translation along the circle S^1; this is a 'trivial', or 'flavor'-vector). Two others ('phase'-vectors) make 2π-rotation along the circle.

In fact, the field (12) has also symmetry $SO_2\{1,4\}$, and this feature restricts possible directions of 'flavor'-vector (two 'flavors' are possible; the $P_2\{1,4\}-$symmetry (this is the π-rotation of x^1, x^4) gives the same effect). The other interesting observation is that the equipped circle can be located also at the stationary points of SO_2-symmetry (this increases the number of 'flavors').

8 Quasi-charges and their morphisms (in $5D$, ie $m = 4$)

If $G \subset SO_4$, the QC-group has two isomorphous parts, left and right: $\Pi(G) = \Pi_{(l)}(G) + \Pi_{(r)}(G)$. The Table 1 describes quasi-charge groups for $G \subset G_0 = (O_3 \times P_4) \cap SO_4$ (P_4 is spatial inversion, the 4-th coordinate is the extra dimension of G_0-symmetric expanding cosmological background). 'Quasi-particles', which symmetry includes P_4, seem to be true neutral (neutrinos, Higgs particles, photon).

One can assume further that an hadron bag is a specific place where G_0-symmetry does not work, and the bag's symmetry is isomorphous to O_4. This assumption can lead to another classification of quasi-solitons (some doubling the above scheme), where self-dual and anti-self-dual one-parameter groups take place of SO_2-group. The total set of quasi-particle parameters (parameters of equipped 1-manifold (loop) plus parameters of group) for (anti)self-dual groups, $G(4,2) \times RP^2$, is larger than the analogous set for groups $SO_2 \subset G_0$, which is just $O_3 \times G(3,1) = RP^2$. If the number of 'flavor'-parameters (which are not degenerate and have some preferable particular values; this should be sensitive to discreet part of G – at least photons have the same flavor) is the same as in the case of 'white' quasi-particles, the remaining parameters (degenerate, or 'phase') can give room for ‘color’ (in addition to spin). So, perhaps one might think about ‘color neutrinos’ (in the context of pomeron, and baryon spin puzzle), ‘color W, Z, and Higgs’ (another context – *B*-mesons).

Note that in this picture the very notion of quasi-particle depends on the background symmetry (also to note: there are no ‘quanta of torsion’ or ‘quanta of curvature’ per se[1]). On the other hand, large clusters of quasi-particles (matter) can disturb the background, and waves of such small disturbances (with wavelength larger than the thickness L, perhaps) can be generated as well (but these waves do not carry (quasi)charges, that is, are *not quantized*).

9 Coexistence: phenomenological ‘quantum fields’ on classical background

The non-linear, particle-like field configurations with quasi-charges (that is, quasi-particles) should be very elongated along the extra-dimension (all of the same size L), while being small sized along usual dimensions, $\lambda \ll L$. The motion of such a spaghetti-like quasi-particle should be very complicated and stochastic due to ‘strong’ imponderable noise, such that different parts of spaghetti are coming their own paths. At the same time, quasi-particle can acquire ‘its own’ energy–momentum – due to scattering of ponderable waves (which wave-vectors are almost tangent to usual $3D$ (sub)space); so, it seems that scattering amplitudes[2] of those spaghetti’s parts which have the same $3D$–coordinates can be summarized thus giving an auxiliary, secondary field.

So, the imponderable waves provides stochasticity (of motion of spaghetti’s parts), while the ponderable waves ensure superposition (with secondary fields). Phenomenology of secondary fields could be of Lagrangian type, with positive energy acquired by quasi-particles, – that to ensure the stability (of all the waveguide with its infill – with respect to quasi-particle production; the least action principle has deep concerns with Lyapunov stability and is deducible, in principle, from the path integral approach).

10 ‘Plain’ R^2 gravity on very thick brane and change in the Newton’s Law of Gravitation

Let us start with 4d (from 5D) bi-Laplace equation with a δ-source [as weak field, non-relativistic (stationary) approximation (it is assumed that ‘mass is possible’) for R^2-gravity (5)] and its solution (R is 4d radius):

$$\Delta^2\varphi = -\frac{a}{R^3}\,\delta(R);\;\; \varphi(R^2) = \frac{a}{8}\ln R^2 - \frac{b}{R^2} + c; \tag{13}$$

the attracting force between two point masses is $F_{\mathrm{point}} = \frac{a}{4R} + \frac{2b}{R^3}$, where a, b should be proportional to both masses m, M (let $m = M = 1$; c in (13) does not matter).

[1] ‘Quantum of temperature’ (or ‘quantum of irony’) is another oxymoron.

[2] These amplitudes can depend on additional vector-parameters (‘equipment vectors’) relating to differential of field mapping at a ‘quasi-particle center’ – where quasi-charge density is largest (if it has covariant sense).

Now let us suppose that all masses are distributed along the extra dimension with a ‘universal function’, $\mu(p)$, $\int \mu(p)\, dp = 1$. Then the attracting (gravitation) force takes the next form [see (13); r is usual 3d distance]:

$$F(r) = \frac{d}{dr} \iint_{-\infty}^{\infty} \varphi(r^2 + (p-q)^2)\, \mu(p)\, \mu(q)\, dp\, dq = \frac{ar}{4} V - bV', \qquad (14)$$

$$\text{where } V(r) = \iint \frac{\mu(p)\, \mu(q)\, dp\, dq}{r^2 + (p-q)^2}.$$

Note that $V(r)$ can be restored if $F(r)$ is measured.

Taking $\mu_1(p) = \pi^{-1}/(1+p^2)$ (typical scale along the extra dimension is taken as unit, $L = 1$; it seems that L should be greater than ten AU), one can find

$$rV_1(r) = \frac{1}{2+r}, \text{ and } F(r) = \frac{a}{8+4r} + \frac{2b(1+r)}{r^2(2+r)^2};$$

$$\text{or (now } L \neq 1) \; F(r) = \frac{1}{r^2} + \frac{r}{2L(2L+r)^2}, \;\text{ where } a = b = \frac{2}{L^2},\; r \to \frac{r}{L}.$$

Fig. 1, curve (a) shows $\delta F = F - 1/r^2$ (more exactly, this is the dimensionless deviation from the Newton’s Law $(F/F_N - 1)L^2/r^2$ as function of r/L); the curves (b), (c) correspond to $\mu_2 = 2\pi^{-1}/(1+p^2)^2$, $\mu_3 = 2\pi^{-1} p^2/(1+p^2)^2$, resp.; residues help to find $rV_2 = (10+6r+r^2)/(2+r)^3$, $rV_3 = (2+2r+r^2)/(2+r)^3$; a/b is also chosen that to ensure $\delta F(0) = 0$. We see that in principle this theory

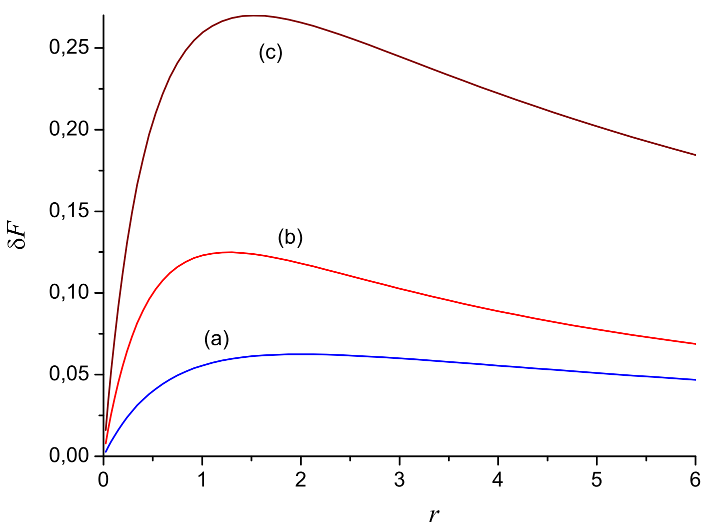

Fig. 1 Deviation $\delta F = F - 1/r^2$ for different $\mu(p)$, see Eq. (14) and text below.

can explain galaxy rotation curves,

$$v^2(r) \sim rF \xrightarrow{r\to\infty} \text{const},$$

without need for Dark Matter or MOND [2]; about rotation curves and DM see [7]; they are looking for DM in Solar system too, [8,9].

Q: Can the ‘coherence of mass’ along the extra dimension be disturbed ? (the flyby anomaly, the Pioneer anomaly [10,11]).

11 Single photon experiment (that to feel huge extra dimension), and Conclusion

Today, many laboratories have sources of single (heralded) photons, or entangled bi-photons (say, for Bell-type experiments [12]); some students can perform laboratory works with single photons, having convinced on their own experience that light is quantized (the Grangier experiment).

It is being suggested a minor modification of the single (polarized) photon interference experiment, say, in a Mach-Zehnder fiber interferometer with ‘long’ (the fibers may be rolled) enough arms. The only new element is a fast-acting shutter placed at the beginning of (one of) the interferometer’s arms (the closing-opening time of the shutter should be smaller than the flight time in the arms). For example, a fast electro-optical modulator in combination with polarizer (or a number of such combinations) can be used with polarized photons.

Both Quantum mechanics (no particle’s ontology) and Bohmian mechanics (wave-particle double ontology) exclude any change in the interference figure as a result of separating activity of such a fast shutter (while the photon’s ‘halves’ are making their ways to the place of a meeting). However, if a photon has non-local spaghetti-like ontology (along the extra dimension) and fragments of this spaghetti are moving along both arms at once, then the shutter should tear up this spaghetti (mainly without photon absorption), tear out its fragments (which will dissolve in ‘zero-point oscillations’). Hence, if the absorption factor of the shutter (the extinction ratio of polarizer) is large enough, the 50/50-proportion (between the photon’s amplitudes in the arms) will be changed and *a significant decrease of the interference visibility should be observed.*

Quantum mechanics is everywhere (where we can see, of course), and, so, non-linear 5*D*-field fluctuations, looking like spaghetti-anti-spaghetti loops, should exist everywhere. This omnipresence can be related to the universality of ‘low-level heat death’, restricted by the presence of topological quasi-solitons – some as the 2*D* computer experiment by Fermi, Pasta, and Ulam, where the process of thermalization was restricted by the existence of solitons. (See once more the sections 5–9 (and [4]) for arguments in favor of phenomenological (quantized) ‘secondary fields’ accounting for topological quasi-charges and obeying superposition, path integral and so on.)

AP, at least at the level of its symmetry, seems to be able to cure the gap between the two branches of physics – General Relativity (with coordinate diffeomorphisms) and Quantum Mechanics (with Lorentz invariance). Most people give all the rights of fundamentality to quanta, and so, they are trying to quantize gravity, and the very space-time (probing loops, and strings, and branes; see also the warning polemic by Schroer [13]). The other possibility is that quanta have the very specific phenomenological origin relating to topological (quasi)charges (and like any other phenomenology, the quantum one has its own boundary of applicability).

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