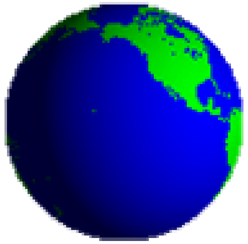


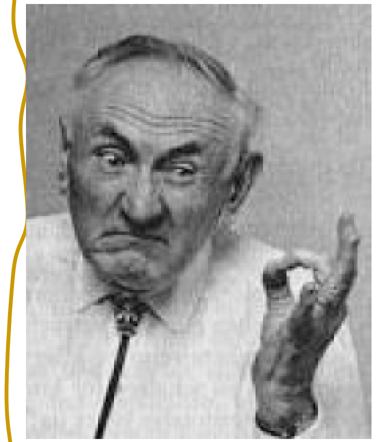
Extra-solar scale change in Newton's Law and 5D R^2-gravity

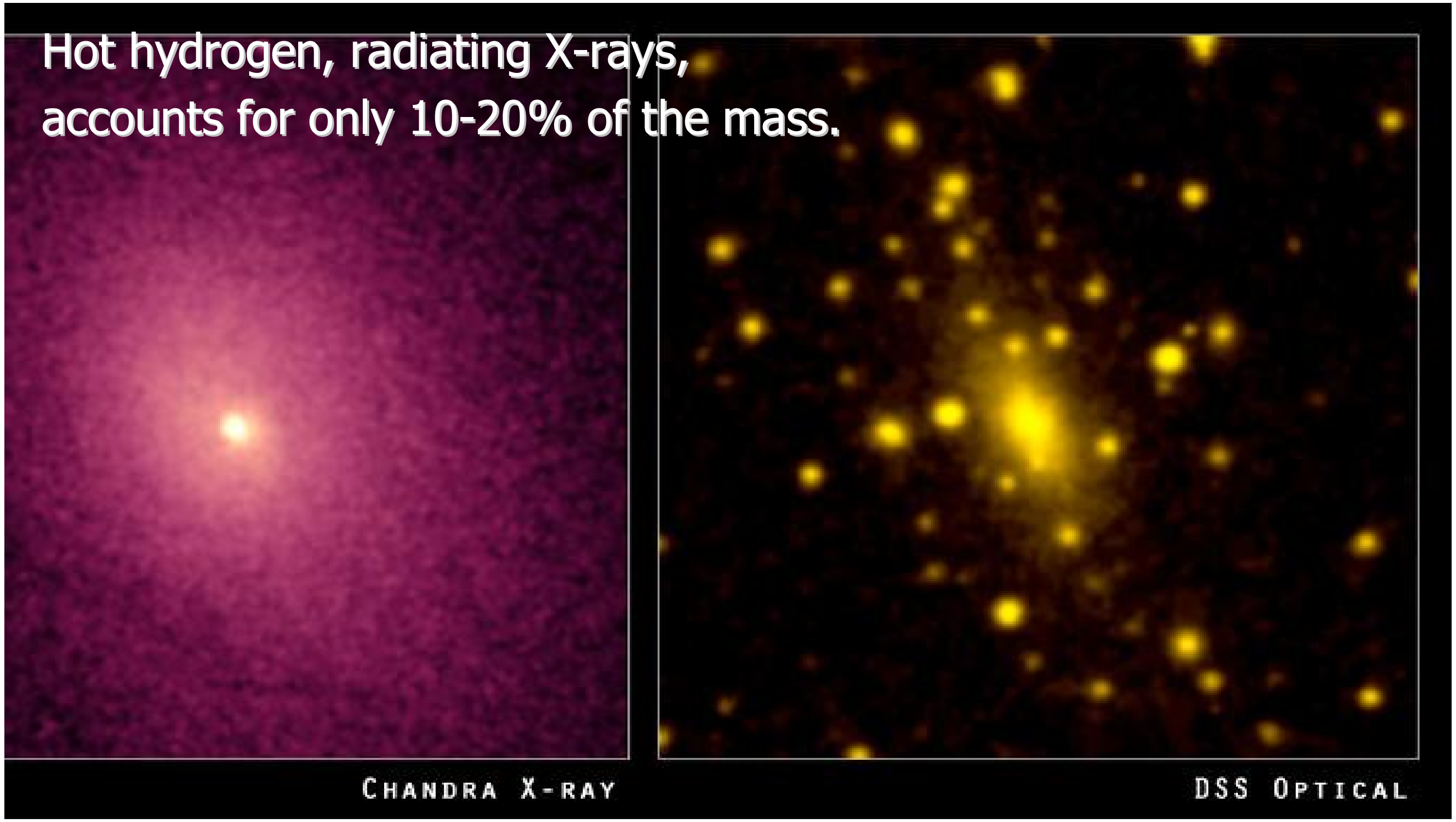
from General Relativity to Absolute Parallelism

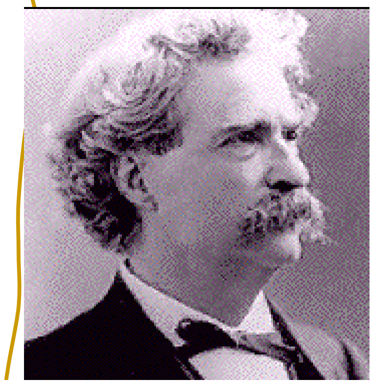
I L Zhogin
SSRC, Novosibirsk
4 July, PIRT-2007

Mass deficit – dark matter ?

Fritz Zwicky computed the mass of the Coma galaxy cluster (in 1932) and found great deficit of luminous mass





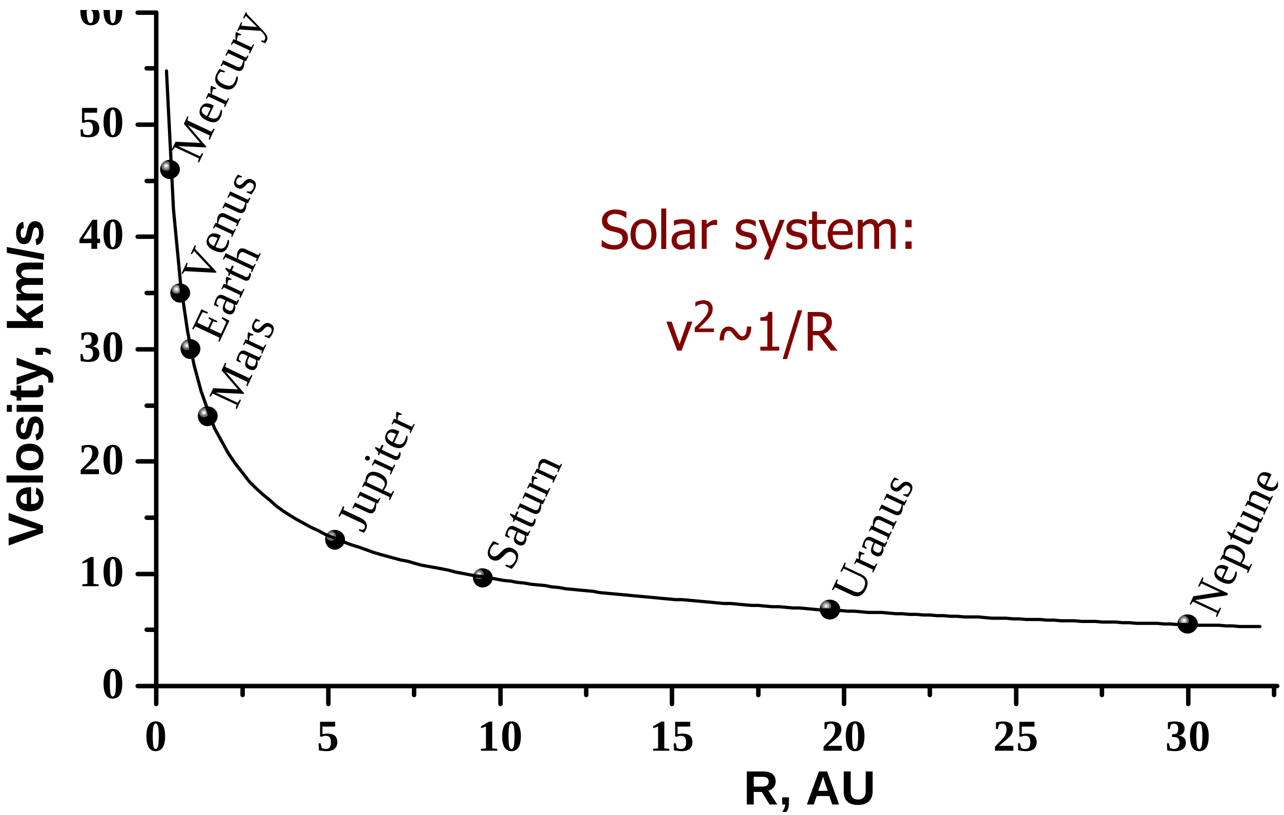


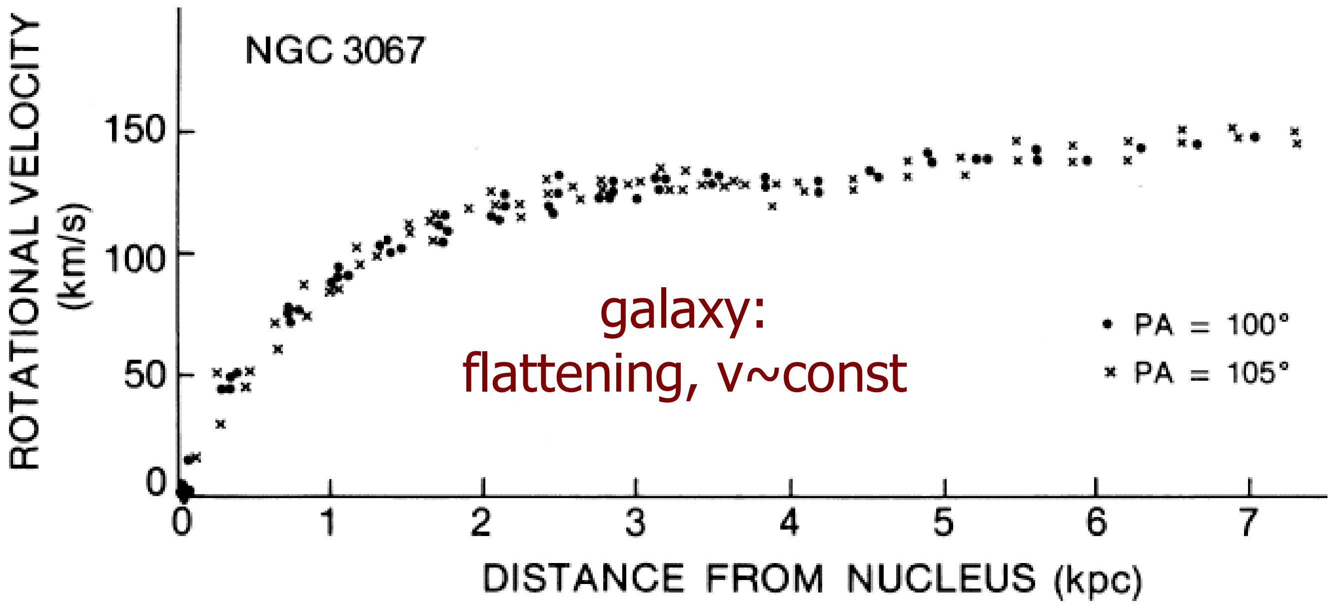
What gets us into trouble is not what we don't know.
It's what we know for sure that just aint so. - Mark Twain

(www.slac.stanford.edu/grp/th/mpeskin/Yale1.pdf
www.astro.umd.edu/~ssm/mond/McGaugh_EdinburghApr06.pdf)

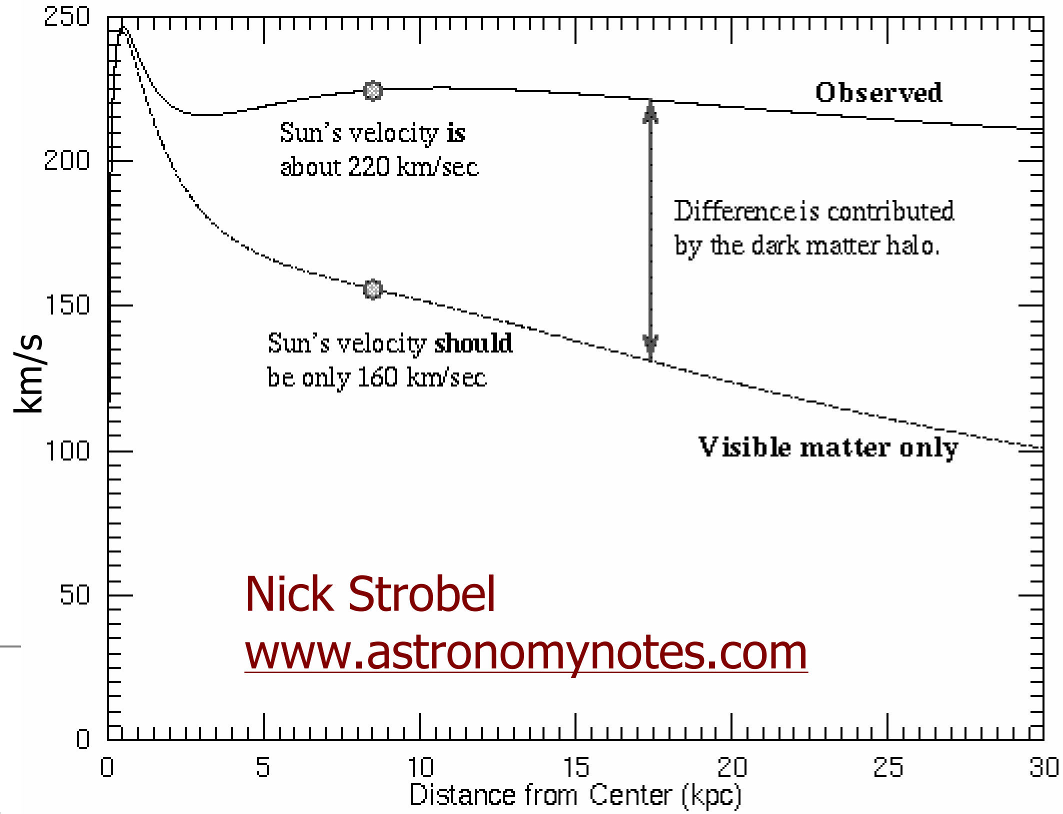
Dark matter ?

Vera Rubin



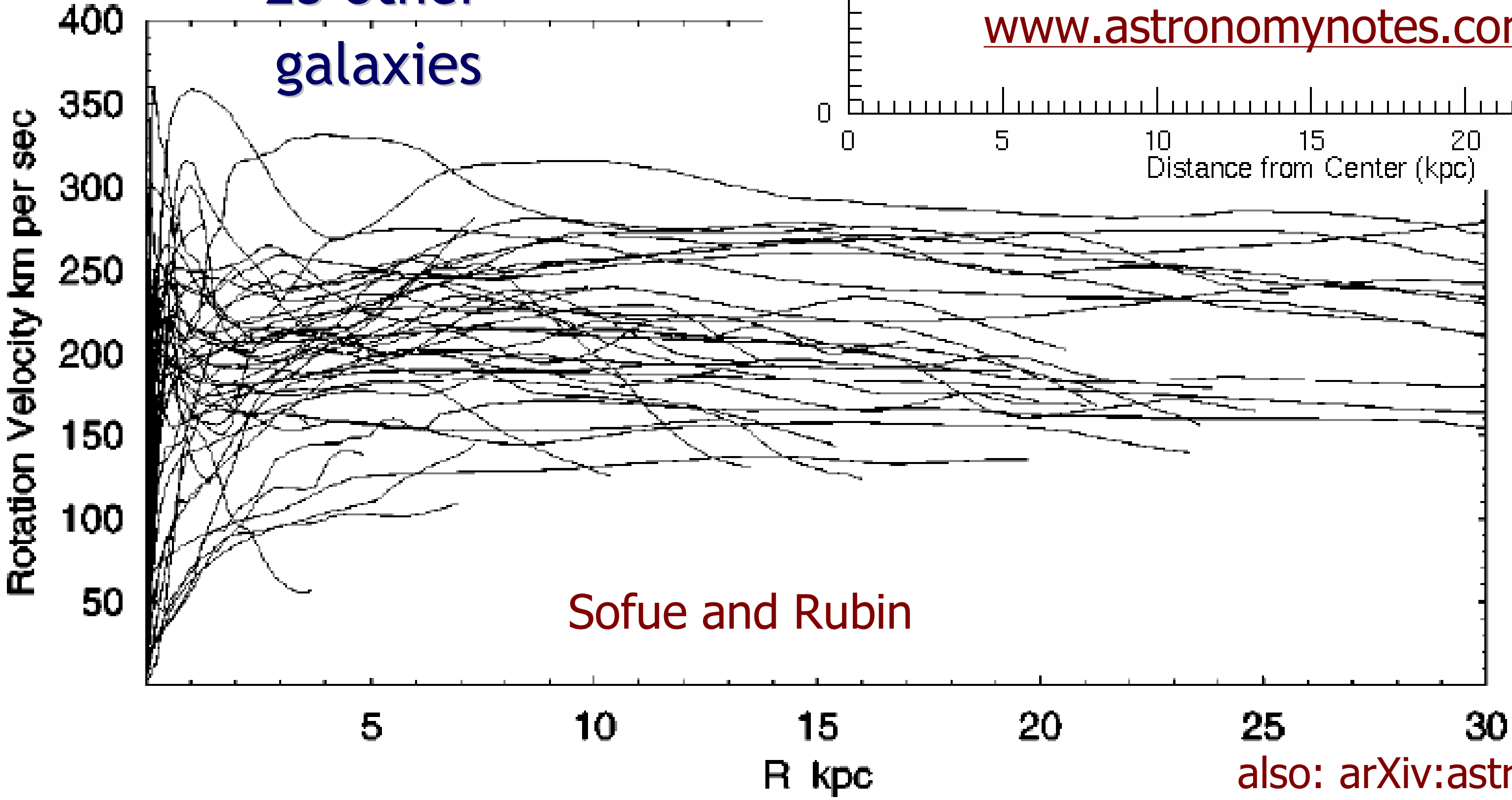


Milky Way



Nick Strobel
www.astronomynotes.com

25 other galaxies



Sofue and Rubin

also: arXiv:astro-ph/0010594

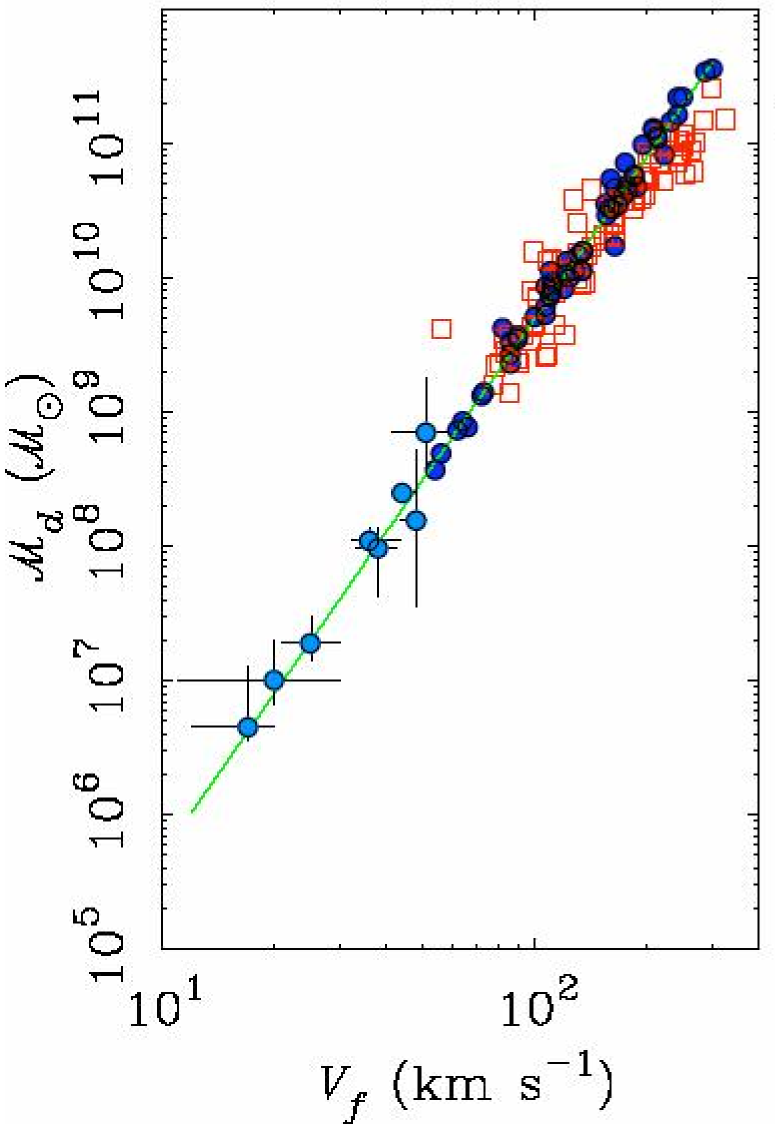
Observations vs DM

Renzo's Rule:

"When you see a feature in the light, you see a corresponding feature in the rotation curve."

The distribution of mass is coupled to the distribution of light.

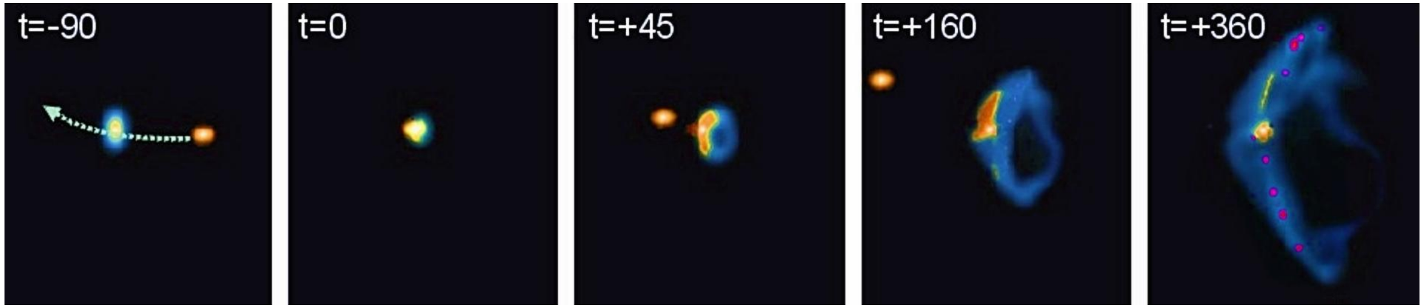
Explaining the data with dark matter always leads to fine-tuning problems



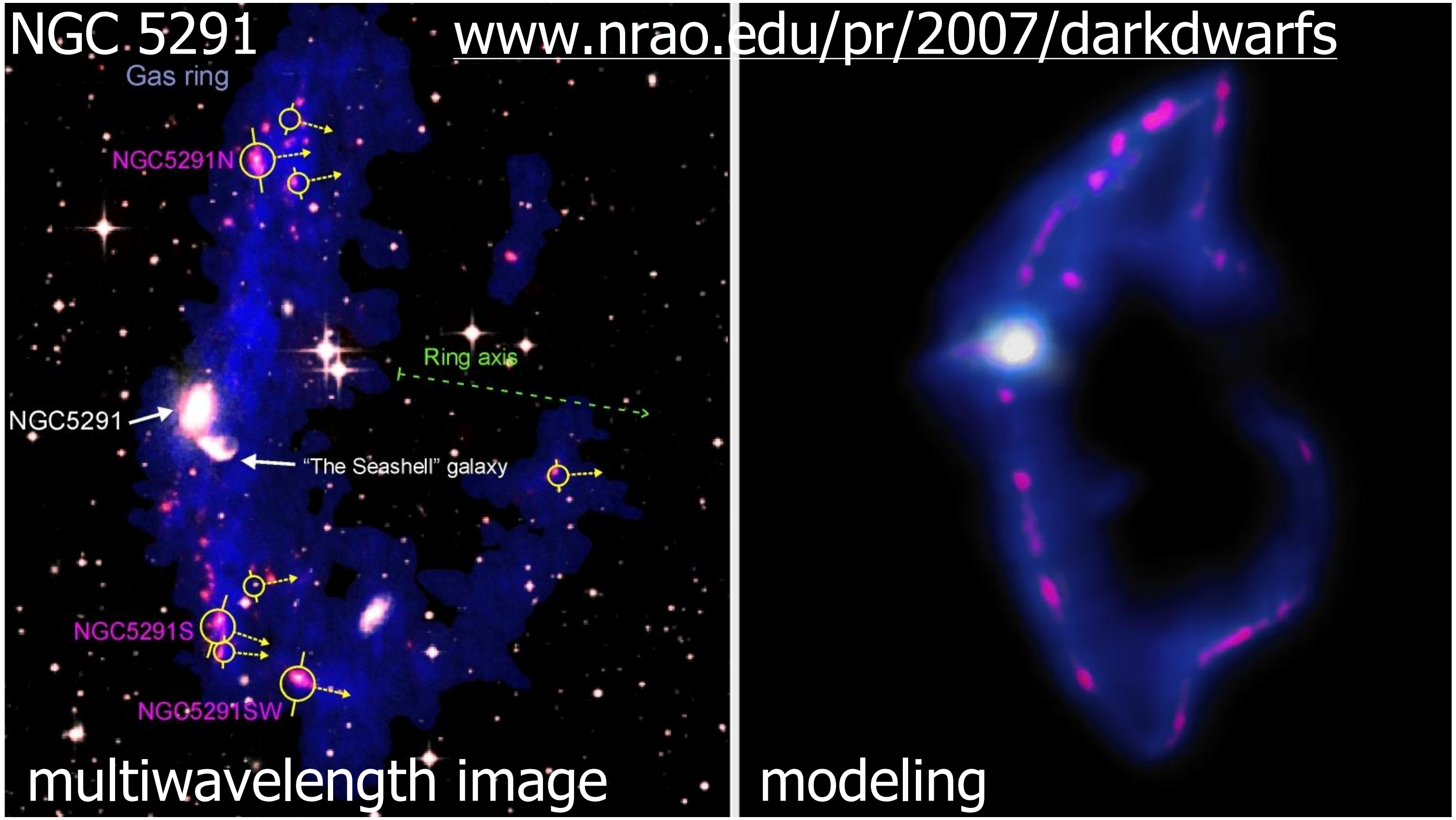
Tully-Fisher relation between luminosity (or mass) and rotation speed. The distribution of mass is coupled to the distribution of light.

is L~ M_b ?

Colliding galaxies



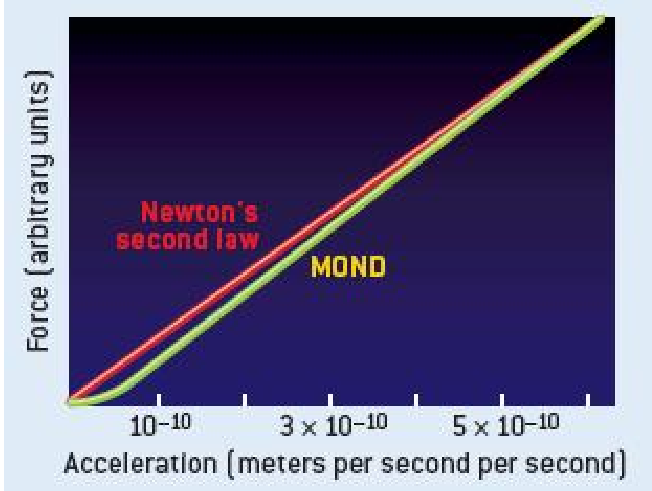
`Recycled' dwarf galaxies formed from debris of a collision of larger galaxies unexpectedly contain unseen matter (VLA radio telescope)



www.astro.umd.edu/~ssm/mond/

Alternative gravity ?

Some people consider DM as a new ether and try to modify gravity or (some of) Newton's law(s)



MOND – modified Newton dynamics

Moti Milgrom

Stacy McGaugh

for $a >> a_o$, $a \Rightarrow g_N$

for $a << a_o$, $a \Rightarrow \sqrt{(g_N a_o)}$

where

$$g_N = GM/R^2$$

MOND does very well in very many tests, but we need good (generally covariant) explanation of this

Experiment with torsion balance shows no deviation from Newton's second law down to $5 \cdot 10^{-14}$ m/s^2

Phys Rev Lett. 98 150801 (2007)

Plain RG-gravity in 5D

extra gravity instead of extra mass

Easy way to change

$$\frac{1}{r^2} \longrightarrow \frac{1}{r}$$

$$\sqrt{-g}\, R_{\epsilon\tau} G^{\epsilon\tau}$$

Lagrangian density for RG-gravity (Ricci and Einstein tensor)

$$\downarrow$$

$$G^{\mu\nu}{}_{;\lambda;\tau} g^{\lambda\tau} + G_{\epsilon\tau}(2R^{\epsilon\mu\tau\nu} - \tfrac{1}{2} g^{\mu\nu} R^{\epsilon\tau}) = T^{\mu\nu}$$

forth-order equation

$$\downarrow$$

$$\Delta^2 \varphi = -\frac{a}{R^3}\delta(R)$$

4d space; point mass (week field, non-relativistic)

$$\downarrow$$

$$\varphi(R^2) = \frac{a}{8}\ln R^2 - \frac{b}{R^2}\ (+c); \quad F_{\text{point}} = \nabla\varphi = \frac{a}{4R} + \frac{2b}{R^3}$$

Prolonged equation of AP: $\mathbf{E}_{(\mu\nu);\lambda}{}^{;\lambda} = 0$

5D RG-gravity

$$\frac{1}{r^2} \longrightarrow \frac{1}{r}$$

masses are distributed along
the extra dimension with a function

$$\int \mu(p)\,dp = 1$$

$$V(r) = \iint \frac{\mu(p)\,\mu(q)\,dp\,dq}{r^2 + (p-q)^2}$$

$$F(r) = \frac{d}{dr}\iint_{-\infty}^{\infty} \varphi(r^2 + (p-q)^2)\,\mu(p)\,\mu(q)\,dp\,dq = \frac{a\,r}{4}V - b\,V'$$

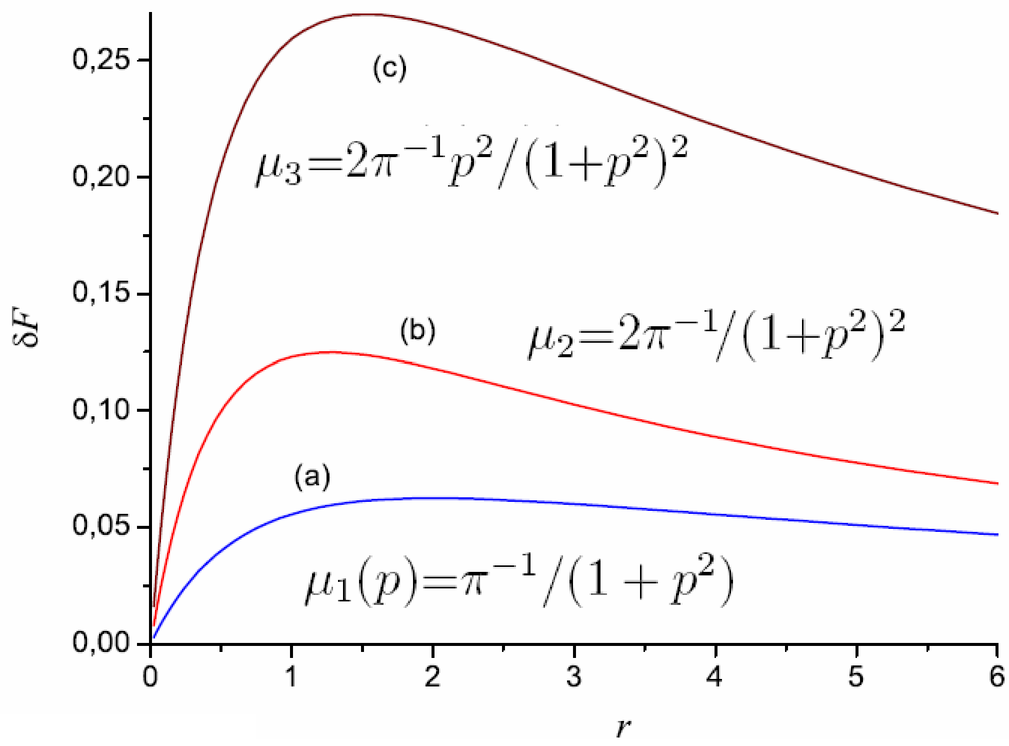
$$\mu_1(p) = \pi^{-1}/(1+p^2)$$

$$F(r) = \frac{a}{8+4r} + \frac{2b(1+r)}{r^2(2+r)^2}$$

$$a = b = 2$$

$$F(r) = \frac{1}{r^2} + \frac{r}{2(2+r)^2}$$

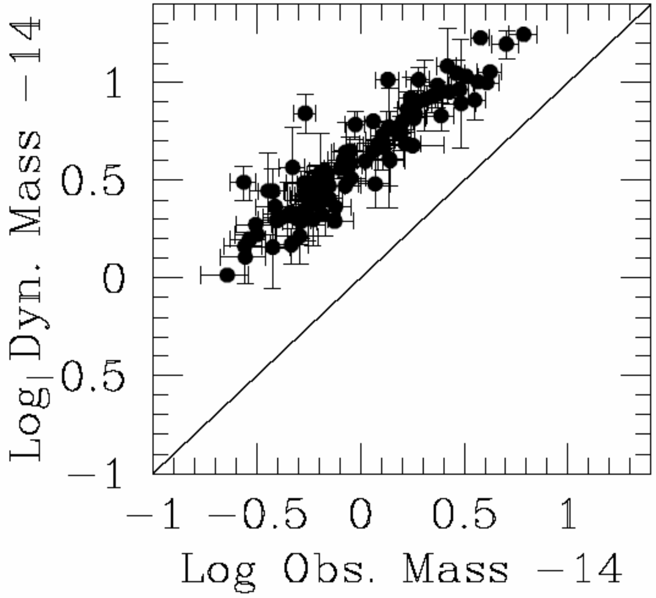
Newton $\frac{1}{r^2} \longrightarrow \frac{1}{r}$ MOND

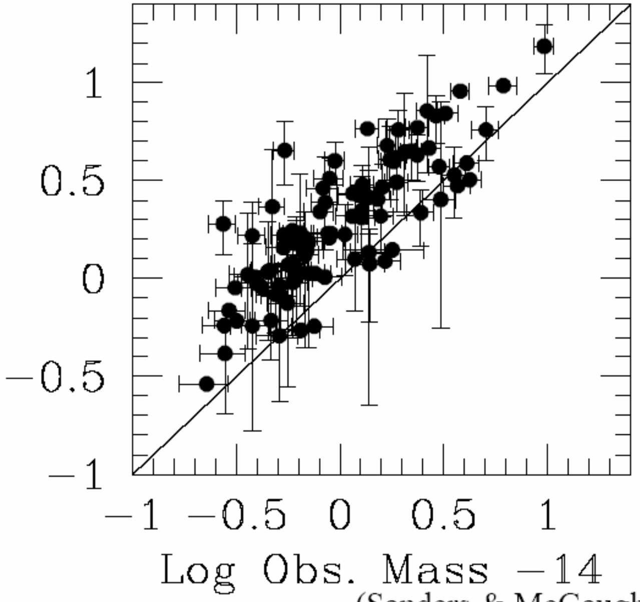


dimensionless deviation from the Newton's Law
$(F/F_N - 1)L^2/r^2$ as a function of r/L

Cluster of galaxies

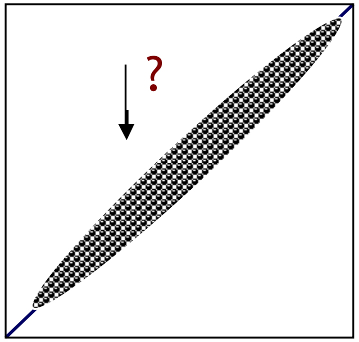
Some problem with TF-relation





$V^2 \propto M \quad (R >> L_4)$

RG-gravity – more gravity for the same amount of matter



How does it work ?

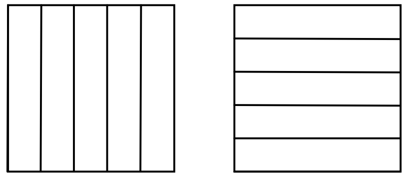
Spaghetti-like particles of matter behave like large Brownian particles immersed into invisible stochastic substrate

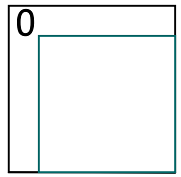
Interesting features of Absolute Parallelism

- High symmetry of equation (+ irreducibility) includes symmetries of both SR and GR; high symmetry of AP equations facilitates the check of its formal integrability

$$\tilde{h}^a{}_\mu(\tilde{x}) = \kappa\,\sigma^a{}_b h^b{}_\nu(x)\frac{\partial x^\nu}{\partial \tilde{x}^\mu} \qquad \kappa > 0,\ \sigma^a{}_b \in O(1, D{-}1)$$

$E_A(x,f,f', \ldots)=0$ — equation;
s: x,f -> x^*,f^* — symmetry,
if $E(x^*,f^*,f^{*\prime}, \ldots)=0$ too.





- Absence of arising singularities and uniqueness: there is one unique variant of AP (non-Lagrangian), with the unique D, D=5, which solutions are free of arising singularities. No room for changes !
- Topological features of non-linear sigma-model: field configurations with topological charge and/or quasi-charge (quasi-solitons)
- Energy-momentum tensor (positive energy; but absence of EM-pseudotensor): conservation laws arise in presence of symmetries (Killing vectors), or in weak field; but not all `polarizations' contribute to energy (powerless waves) — how to quantize ?
- Instability of trivial solution (growing powerless waves); non-stationary O_4-symmetric solutions (single waves) as more appropriate expanding cosmological background — to be filled with stochastic waves and topological quasi-solitons

Covariants..

covariant differentiation with Levi-Civita connection

metric - $g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu, \quad \eta_{ab}, \eta^{ab} = \text{diag}(-1, 1, \ldots, 1)$

`main' tensor - $\Lambda_{a\mu\nu} = h_{a\mu,\nu} - h_{a\nu,\mu} = 2h_{a[\mu;\nu]} \quad (g_{\mu\nu;\lambda} \equiv 0)$

and its irred. parts - $S_{abc} = 3\Lambda_{[abc]} = \Lambda_{abc} + (abc), \quad \Phi_a = \Lambda_{b\mu\nu} h_b{}^\mu h_a{}^\nu$

important tensor (!) - $f_{\mu\nu} = 2\Phi_{[\mu;\nu]} = \Phi_{\mu;\nu} - \Phi_{\nu;\mu}; \quad f_{[\mu\nu;\lambda]} \equiv 0$

identity - $\Lambda_{a[\mu\nu;\lambda]} \equiv 0 \quad \Lambda_{abc,a} + f_{bc} \equiv 0 \quad ({}_{,a} = {}_{;\mu} h_a{}^\mu)$

Riemannian curvature - $R_{a\mu\nu\lambda} = 2h_{a\mu;[\nu;\lambda]}; \; h_{a\mu} h_{a\nu;\lambda} = \frac{1}{2} S_{\mu\nu\lambda} - \Lambda_{\lambda\mu\nu}$

..and covariant eqns

$$\mathbf{E}_{a\mu} = \Lambda_{a\mu\nu;\nu} = 0; \; \mathbf{E}_{a\mu;\mu} = \Lambda_{a\mu\nu;\nu;\mu} \equiv 0$$

$$\mathbf{G}_{\mu\nu} = 2\mathbf{E}_{(\mu\nu)} = \Lambda_{\mu\nu\lambda;\lambda} + \Lambda_{\nu\mu\lambda;\lambda} +$$

$$+\sigma(\Phi_{\mu;\nu} + \Phi_{\nu;\mu} - 2g_{\mu\nu}\Phi_{\lambda;\lambda}) + W_{(\mu\nu)}(\Lambda^2) = 0$$

← symmetrical part

$$\mathbf{H}_{\mu\nu} = 2\mathbf{E}_{[\mu\nu]} = S_{\mu\nu\lambda;\lambda} + \tau f_{\mu\nu} + W_{[\mu\nu]}(\Lambda^2) = 0$$

← skew-symmetrical part

$$\mathbf{G}_{\mu\nu;\nu} = (\sigma - 1)(f_{\mu\nu;\nu} - J^{(1)}_\mu(\Lambda'\Lambda, \Lambda^3)) = 0$$

$$\mathbf{H}_{\mu\nu;\nu} = \tau(f_{\mu\nu;\nu} - J^{(2)}_\mu(\Lambda'\Lambda, \Lambda^3)) = 0$$

← prolonged equations

$$J^{(1)}_\mu = J^{(2)}_\mu \to \quad \tau\mathbf{G}_{\mu\nu;\nu} + (1-\sigma)\mathbf{H}_{\mu\nu;\nu}$$

← identity required for compatibility

Co-singularities and unique equation

determinant and minors (multi-linear expressions of co-frame)

$$h = \det h^a{}_m,\ \partial h/\partial h^a{}_m = h h_a{}^m = \binom{m}{a}$$

$$\binom{mn}{ab} = \partial^2 h/\partial h^a{}_m \partial h^b{}_n = h\left(h_a{}^m h_b{}^n - h_a{}^n h_b{}^m\right)$$

2-minor (co-rank 2)

$$h^2 \Lambda_a{}^{mn}{}_{,n} + \mathbf{L} = h_{ae,tn} \underbrace{h^2\left(g^{em}g^{tn} - g^{en}g^{tm}\right)}_{[me,nt] = \frac{1}{2}\binom{mn}{ab}\binom{et}{ab}} + (h'^2)$$

$$h^a{}_m = \begin{pmatrix} 1 & & & \\ & \mathbf{O} & & \\ & & 1 & \\ & & & 0 \end{pmatrix}$$

Two special cases (when two parts should be treated separately)

$$h^2 \mathbf{G}^{mn} = \left\{(-g)\left(g^{em}g^{tn} - g^{en}g^{tm}\right)\right\}_{,et} + (g'^2) = [mn,et,ab]\left(g_{ab,et} + \Gamma^r{}_{ab}\Gamma_{r,et}\right) = 0$$

$$h^2 \mathbf{H}_{ab} = 0$$

GR

1-minor x 2-minor

$$h^2 \mathbf{G}_{ab} = 0$$

$$h^2 \mathbf{H}^{mn} = h^2 S^{mnl}{}_{;\lambda} \propto \begin{pmatrix} m & n & l \\ a & b & c \end{pmatrix}\begin{pmatrix} a & b \\ b & c \end{pmatrix} h_{aa,bl} + h'^2 = 0$$

Unique eqn (UE, antipode of GR)

$$L_{amn;n} - \tfrac{1}{3} f_{am} - \tfrac{1}{3} L_{amn}\Phi_n = 0,\quad L_{amn} = \Lambda_{amn} - S_{amn} - \tfrac{2}{3} h_{a[m}\Phi_{n]},\quad L_{aan} = \tfrac{4-D}{3}\Phi_n$$

$$2\mathbf{E}_{[\nu\mu]} = S_{\mu\nu\lambda;\lambda} = 0,\ \ \mathbf{E}_{[\nu\mu];\nu} \equiv 0;\ \ \mathbf{E}_{\mu\mu} = \mathbf{E}_{a\mu} h^\mu_b \eta^{ab} = \frac{4-D}{3}\Phi_{\mu;\mu} + (\Lambda^2) = 0.$$

D=4 - forbidden

Contra-singularities and unique *D*

$$H_a{}^\mu = h^{1/D_*} h_a{}^\mu;\ H = \det H^a{}_\mu,\ \ h_a{}^\mu = H^{1/(D-D_*)} H_a{}^\mu$$

Contra-variant density of some weight;
D_* depends on equation (D_* = 2 for GR)

$$H_a^{\ m} = \begin{pmatrix} 1 & & & \\ & \mathbf{O} & & \\ & & 1 & \\ & & & 0 \end{pmatrix}$$

3-linear form of unique equation
(D_*=4 — forbidden dimension):

$$\left(H_a{}^\mu{}_{,\nu\lambda} H_b{}^\nu - H_b^\mu{}_{,\nu\lambda} H_a{}^\nu - \frac{2}{3} H_a{}^\nu{}_{,\nu\lambda} H_b{}^\mu\right) H_b{}^\lambda + \frac{1}{3} H_b{}^\nu{}_{,\nu\lambda}\left(H_a{}^\mu H_b{}^\lambda + H_a{}^\mu H_b{}^\lambda\right) + H_a{}^\mu{}_{,\nu}\left(H_b{}^\nu H_b{}^\lambda\right)_{,\lambda}$$

$$+ H_b{}^\mu{}_{,\nu}\left(H_b{}^\nu{}_{,\lambda} H_a{}^\lambda - 2H_a{}^\nu{}_{,\lambda} H_b{}^\lambda - H_b{}^\lambda{}_{,\lambda} H_a{}^\nu\right) + \frac{2}{9} H_b{}^\lambda{}_{,\lambda}\left(H_b{}^\nu{}_{,\nu} H_a{}^\mu - H_a{}^\nu{}_{,\nu} H_b{}^\mu\right) = 0$$

this can be useful for phenomenological division $H = H_c + H_{ch} + H_t$

$$h^a{}_m = H^p H^a{}_m\ ,\quad p = 1/(4-D) \to -1 \ \text{ if } \ D = 5$$

$$h^a{}_m = H^{-1} H^a{}_m$$

1-minor of «working» matrix coincides with co-frame; that is, contra-singularity simultaneously implies co-singularity (of very high co-rank)

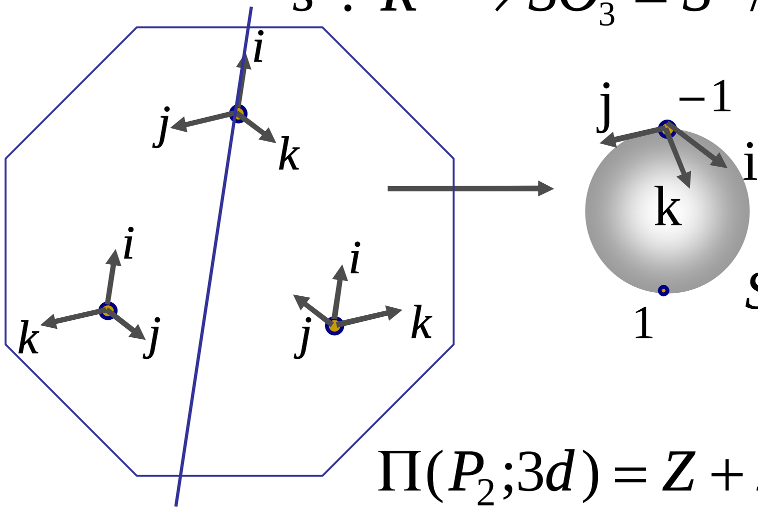
the best choice of *D*: *D*=5 ! !

Topological charges and quasi-charges

$$g_{mn} \Rightarrow h_{mn}\,, \quad h^{a}_{\ m} \Rightarrow s^{\ a}_{\ m} \in SO(1, D-1)$$

$$\Pi(0;3) = p_3(SO_3) = Z$$

$$s\ :\ R^3 \rightarrow SO_3 = S^3/\pm$$



$$\Pi(P_2;3d) = Z + Z$$

$$\text{q}^* = \text{a} \cdot \text{q} \cdot \text{b}^{-1}, \quad |\text{a}| = |\text{b}| = 1$$

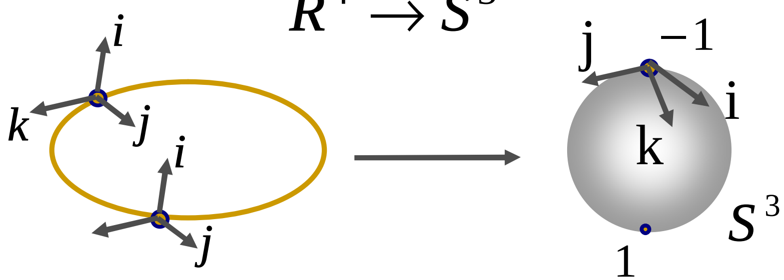
$$\text{q} = x_1 \cdot \text{i} + x_1 \cdot \text{j} + x_1 \cdot \text{k} + x_4$$

$$\uparrow \qquad (\text{f}\,,\text{g})$$

$$s\ :\ R^4 \rightarrow SO_4 = S^3 \times S^3/\pm$$

$$\Pi(0;4) = p_4(SO_4) = Z_2 + Z_2$$

$$R^4 \rightarrow S^3$$



$(\text{f}\,,\text{g})$ $\qquad$ (a, b)

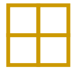
$$s\,(sx) = s\,s\,(x)s^{-1} \quad \forall s \in G \subset SO_3 \times P_4$$

Energy-momentum tensor

$\mathbf{E}_{(\mu\nu)} = 0$ $\quad G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ $\quad \Phi_{(\mu;\nu)}$

Prolonged equation, $\mathbf{E}_{(\mu\nu);\lambda;\lambda} = 0$, can be written as RG-gravity:

$$(-h^{-1}\,\delta(h\, R_{\mu\nu} G^{\mu\nu})/\delta g_{\mu\nu}=)\; G_{\mu\nu;\lambda;\lambda} + G_{\epsilon\tau}(2R_{\epsilon\mu\tau\nu} - \tfrac{1}{2} g_{\mu\nu} R_{\epsilon\tau}) = T_{\mu\nu}(\Lambda'^2, \cdots),\; T_{\mu\nu;\nu} = 0$$

$$T_{\mu\nu} = \tfrac{2}{9}(\tfrac{1}{4} g_{\mu\nu} f^2 - f_{\mu\lambda} f_{\nu\lambda}) + A_{\mu\epsilon\nu\tau}(\Lambda^2)_{,\epsilon\tau} \longleftarrow$$ 

$$f_{mn;n} = -\tfrac{1}{2} S_{mnl}\, f_{nl}\,, \quad f_{[mn;l]} \equiv 0 \quad h_{a[e}\Lambda^a{}_{mn;l]} \equiv 0 \rightarrow S_{[emn;l]} - \tfrac{1}{2}\Lambda_{a[el}\Lambda^a{}_{mn]} \equiv 0$$

This theory does not match GR, but shows `plain' R^2-gravity;

only f-component (three transverse polarizations in D=5) carries D-momentum and angular momentum (`powerful' waves); other 12 polarizations are `powerless', or `weightless' (this is a very unusual feature – impossible in the Lagrangian tradition; how to quantize ?);

f-component feels only metric and S-field which has effect only on polarization of f: S does not enter eikonal equation, and f moves along usual Riemannian geodesic (if background has f=0);

the trace Tμμ can be non-zero ;

pseudo-tensor is lacking

$\mathbf{E}^2_{(\mu\nu)}$ -- trivial “weak Lagrangian” -> $RG+f^2+a_{\mu;\mu}$

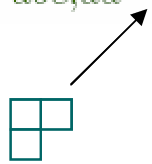
Instability of trivial solution

Linearized equations:

$$3\Lambda_{abd,d} = \Phi_{a,b} - 2\Phi_{b,a} \qquad \Phi_{a,a} = 0 \qquad \Lambda_{a[bc,d],d} \equiv 0 \;\Rightarrow\; 3\Lambda_{abc,dd} = -2f_{bc,a}$$

$$S_{abc,dd} = 0,\;\; \Phi_{a,dd} = 0,\;\; f_{ab,dd} = 0,\;\; R_{abcd,ee} = 0$$

`stable' polarizations



unstable, ie growing polarizations, ~ t

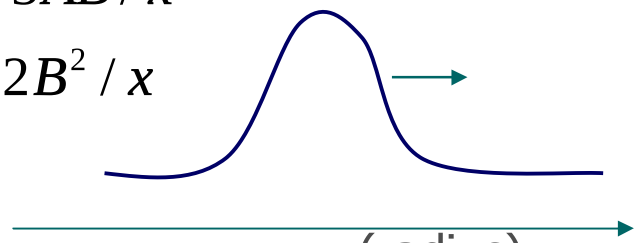
–> we need another background

Spherically-symmetrical solutions

$$h^{a}{}_{m} = \begin{pmatrix} a & bn_i \\ cn_i & en_i n_j + d\Delta_{ij} \end{pmatrix}, \;\; \Delta_{ij} = d_{ij} - n_i n_j, \;\; n_i = x^i / x \qquad f_{mn} = 0, \;\; S_{mnl} = 0$$

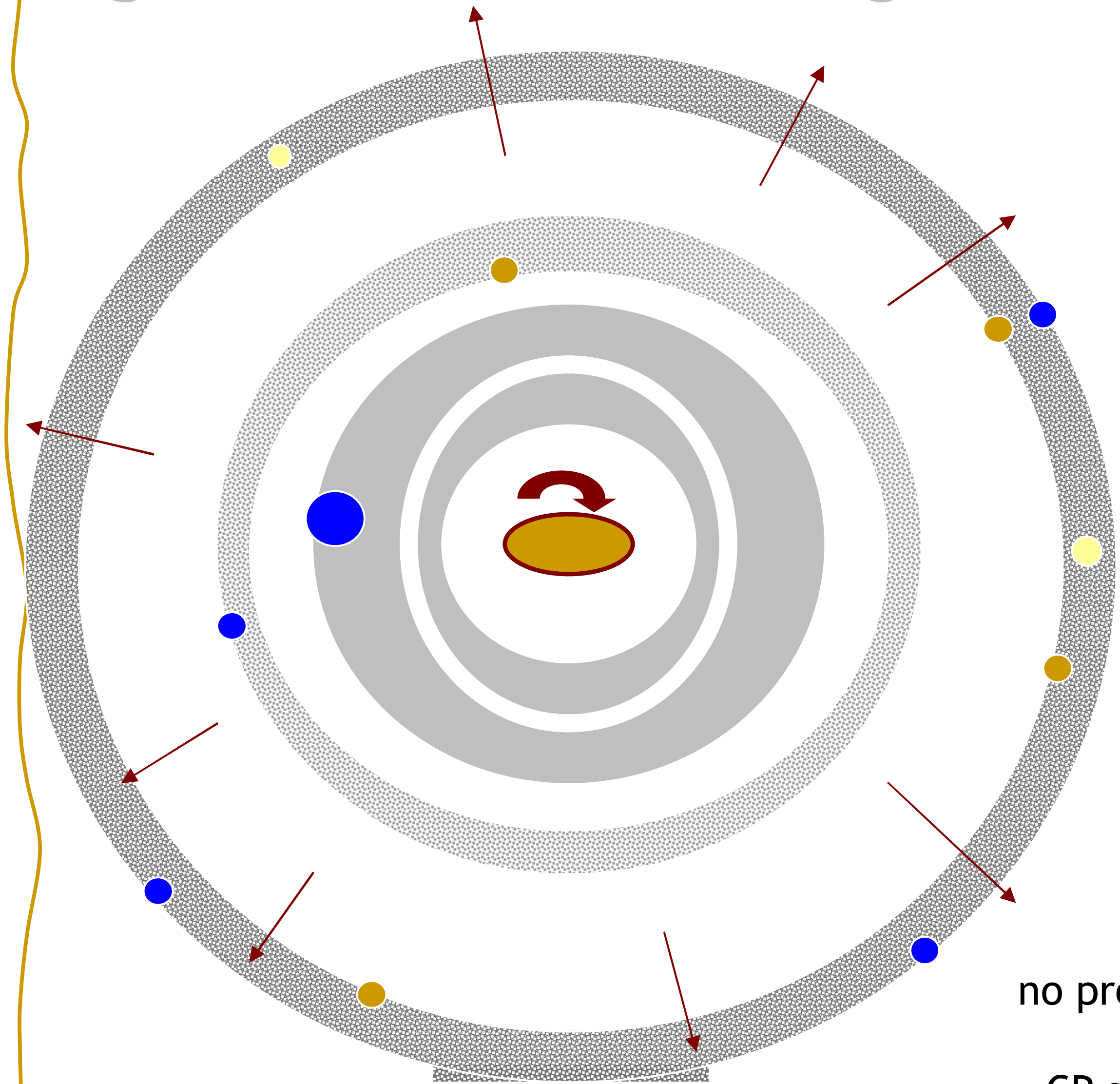
$$b = 0, \quad e \qquad\qquad A^{\bullet} = AB' - BA' + 3AB/x$$

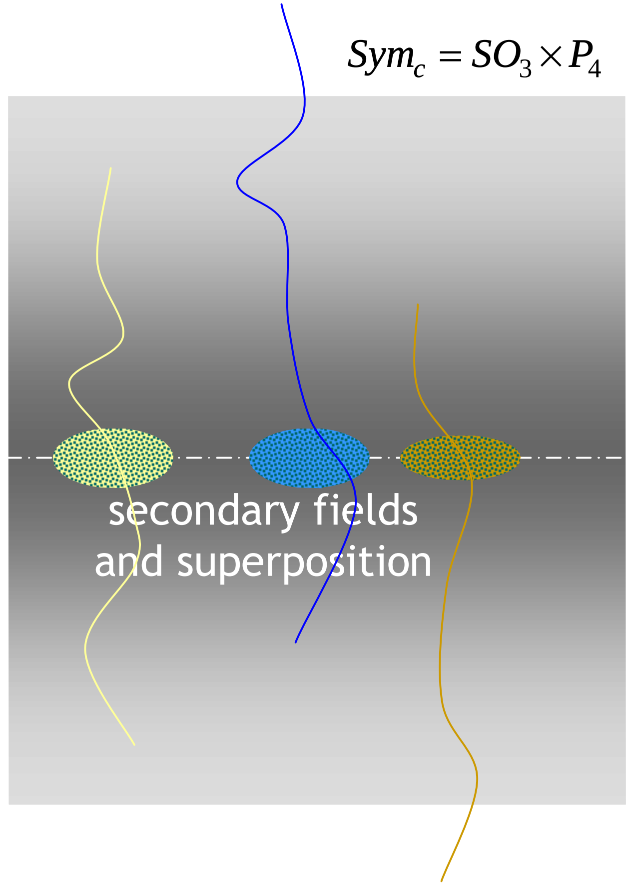
$$A = a/e = e^{1/2}, \quad B = -c/e \qquad B^{\bullet} = AA' - BB' + 2B^2/x$$



Non-stationary O_4-symmetrical solutions exist which have chances to be stable

Model of repeatable big bangs; giant relativistic surfing with O_4-wave





Uniform expansion –
no problems with GR-deacceleration;
Closed Universe;
CP symmetry becomes more exact

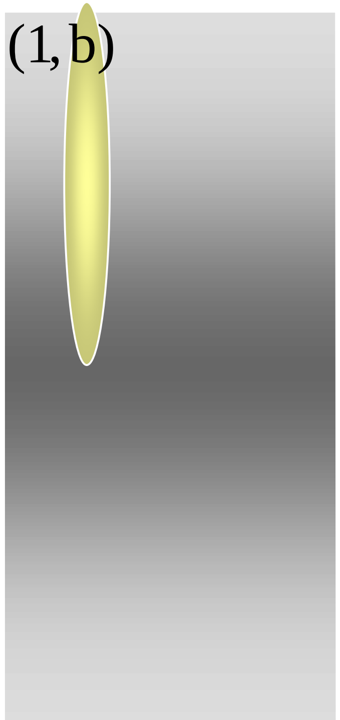
D=5: QC-groups and morphisms

Symmetries SO_2xDiscr.

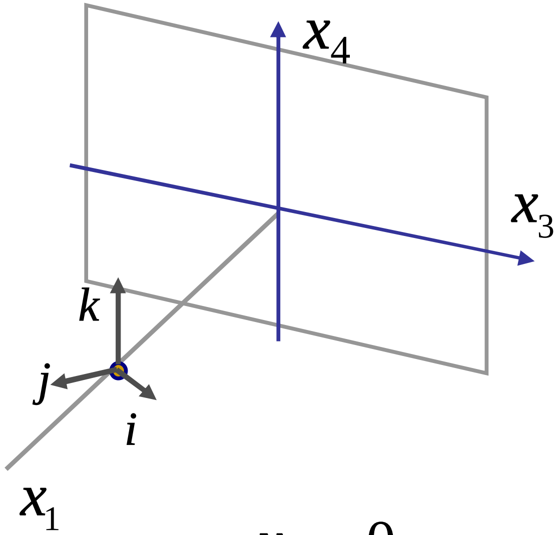
$SO_2^+ = (\mathrm{a},1), \quad SO_2^- = (1,\mathrm{b})$

symmetry of hadron bag $\cong O_4$

symmetry of background $Sym_c = SO_3 \times P_4$



$SO\{1,2\} \times P\{4,3\}$



$x_2 = 0$

Sym	$\Pi_l(Sym) \to \Pi_l(Sym^*)$	
1	Z_2	
$SO\{1,2\}$	$Z_{(e)} \xrightarrow{e} Z_2$	e
$SO\{1,2\} \times P\{0,3\}$	$Z_{(\nu)} + Z_{(H)} \xrightarrow{i,m2} Z_{(e)}$	$\nu^0;\ H^0$
$SO\{1,2\} \times P\{2,3\}$	$Z_{(W)} \xrightarrow{0} Z_{(e)}$	W
$SO\{1,2\} \times P\{0,2\}$	$Z_{(Z)} \xrightarrow{0} Z_{(e)}$	Z^0
$SO\{1,2\} \times P\{0,3\} \times$ $\times P\{2,3\}$	$Z_{(\gamma)} \xrightarrow{0} Z_{(H)}$ $\xrightarrow{0} Z_{(W)}$	γ^0

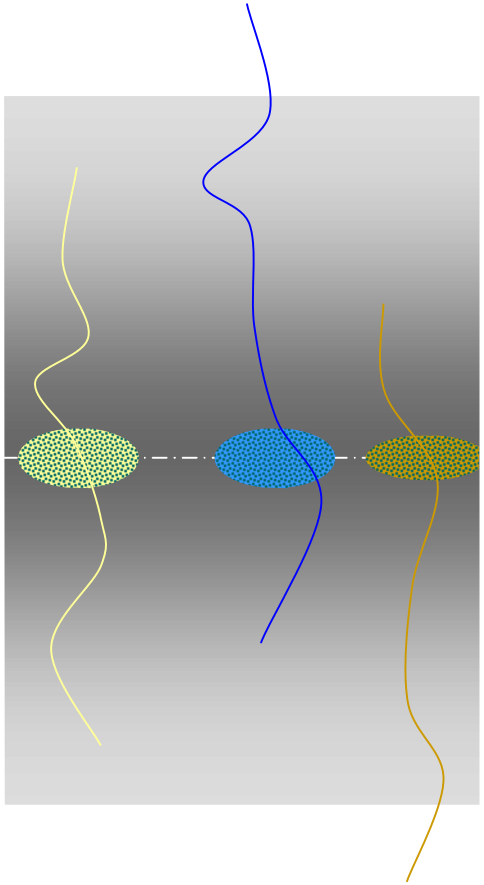
Conclusion

The uniqueness of theory (no room for changes) is a significant and unique advantage (string theory contains at least the string tension as a free parameter)

In spite of insufficient study, some qualitative results or predictions are possible:
Uniform cosmological expansion (no problems of de-acceleration); closed Universe;
CP symmetry becomes more exact with time

This theory predicts not only gravity (RG) but also definite set of `particles' (quasi-solitons; all should carry spin)

A simple modification of experiment with single photon interference can be suggested that to check in a lab the possible existence of the extra dimension (and spaghetti-like origin of particles; arXiv/astro-ph/0704.0857; poster)



Thank you for your attention