Extra-solar scale change in Newton's Law: R^2-gravity and extra-dimension

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The examination of formal integrability (or well-posedness) of generally covariant equations of Absolute Parallelism (and GR as a special case of AP) can be extended into the cases when the co-frame matrix, $h^a{}_\mu$, (or contra-frame density of some weight) is degenerated. This local and covariant test for singularities singles out the only variant of AP (and the only D, D=5) which solutions of general position seem to be free of singularities.

Then some points of the theory are sketched: instability of trivial solution and expanding O_4-symmetrical ones; tensor $T_{\mu\nu}$ (positive energy, but only three polarizations of 15 carry (and angular) momentum; how to quantize ?) and PN-effects; topological classification of symmetric 5D field configurations (alighting on evident parallels with Standard Model' particle combinatorics) and *quantum phenomenology* on expanding classical background; *plain* R^2 gravity on very thick brane and change in the Newton's Law: $\frac{1}{r^2}$ goes to $\frac{1}{r}$ with distance.

At last, an experiment with single photon interference is discussed as the other way to observe very-very long (and very undeveloped) the extra dimension.

1 Co- and contra-singularities in AP and unique $5D$ equation

Absolute Parallelism (AP) has many interesting features: large symmetry group of equations; field irreducibility with respect to this group; vast list of compatible second order equations not restricted to Lagrangian ones.

There is one unique variant of AP (non-Lagrangian, with the unique D; D=5) which solutions of general position seem to be free of arising singularities. The formal integrability test [1] can be extended to the cases of degeneration of either co-frame matrix, $h^a{}_\mu$, (co-singularities) or contra-variant frame (or contra-frame density of some weight), serving as the local and covariant (no coordinate choice) test for singularities of solutions. In AP this test singles out the next equation (and D=5, see [2]; $\eta_{ab} = \mathrm{diag}(-1, 1, \ldots, 1)$, then $h = \det h^a{}_\mu = \sqrt{-g}$):

$$\mathbf{E}_{a\mu} = L_{a\mu\nu;\nu} - \tfrac{1}{3}(f_{a\mu} + L_{a\mu\nu}\Phi_\nu) = 0\,; \tag{1}$$

here $L_{a\mu\nu} = L_{a[\mu\nu]} = \Lambda_{a\mu\nu} - S_{a\mu\nu} - \frac{2}{3}h_{a[\mu}\Phi_{\nu]}$, $\Lambda_{a\mu\nu} = 2h_{a[\mu,\nu]}$, $S_{\mu\nu\lambda} = 3\Lambda_{[\mu\nu\lambda]}$, $\Phi_\mu = \Lambda_{aa\mu}$, $f_{\mu\nu} = 2\Phi_{[\mu,\nu]}$. Coma "," and semicolon ";" denote partial derivative and usual covariant differentiation with symmetric Levi-Civita connection, respectively. One should retain the identities: $\Lambda_{a[\mu\nu;\lambda]} \equiv 0$, $h_{a\lambda}\Lambda_{abc;\lambda} \equiv f_{cb}\,(= f_{\mu\nu}h^\mu_c h^\nu_b)$, $f_{[\mu\nu;\lambda]} \equiv 0$. The equation $\mathbf{E}_{a\mu;\mu} = 0$ gives 'Maxwell-like equation' (for brevity, we omit η_{ab} and $g^{\mu\nu} = h^\mu_a h^\nu_a$ in contractions):

$$(f_{a\mu} + L_{a\mu\nu}\Phi_\nu)_{;\mu} = 0, \ \text{or} \ \ f_{\mu\nu;\nu} = (S_{\mu\nu\lambda}\Phi_\lambda)_{;\nu} \ \ (= -\tfrac{1}{2}S_{\mu\nu\lambda}f_{\nu\lambda}, \ \text{see below})\,. \tag{2}$$

Really (2) follows from the symmetric part, because skewsymmetric one gives just the identity; note also that the trace part becomes irregular (the principal derivatives vanish) if $D = 4$ (forbidden number of dimension):

$$2\mathbf{E}_{[\nu\mu]} = S_{\mu\nu\lambda;\lambda} = 0, \ \ \mathbf{E}_{[\nu\mu];\nu} \equiv 0; \quad \mathbf{E}_{\mu\mu} = \mathbf{E}_{a\mu}h^\mu_b\eta^{ab} = \tfrac{4-D}{3}\Phi_{\mu;\mu} + (\Lambda^2) = 0.$$

The system (1) remains compatible under adding $f_{\mu\nu} = 0$, see (2); this not the case for another covariant, S, Φ, or Riemannian curvature, which relates to Λ as usually: $R_{a\mu\nu\lambda} = 2h_{a\mu;[\nu;\lambda]}$; $h_{a\mu}h_{a\nu;\lambda} = \frac{1}{2}S_{\mu\nu\lambda} - \Lambda_{\lambda\mu\nu}$.

GR is a special case of AP. Using 3-minors (i.e., co-rank 3) of co-metric, $[\mu\nu, \varepsilon\tau, \alpha\beta] \equiv \partial^3(-g)/(\partial g_{\mu\nu}\partial g_{\varepsilon\tau}\partial g_{\alpha\beta})$, and their skew-symmetry, one can write the vacuum equation of GR (the principal derivatives) as follows:

$$0 = 2(-g)G^{\mu\nu} = [\mu\nu, \varepsilon\tau]_{,\varepsilon\tau} + (g'^2) = [\mu\nu, \varepsilon\tau, \alpha\beta](g_{\alpha\beta,\varepsilon\tau} + g^{\rho\phi}\Gamma_{\rho,\varepsilon\tau}\Gamma_{\phi,\alpha\beta}) = [\mu\nu, \varepsilon\tau, \alpha\beta]R_{\alpha\beta\varepsilon\tau}\,. \tag{3}$$

In a similar way, almost all AP equations (with two exceptions) can be written such that 2-minors of co-frame,

$$\begin{pmatrix} \mu\ \nu \\ a\ b \end{pmatrix} = \partial^2 h/\partial h^a{}_\mu \partial h^b{}_\nu = 2!\, h h^\mu_{[a} h^\nu_{b]} \ \ (\text{note that} \ \ [\mu_1\nu_1, \ldots, \mu_k\nu_k] = \tfrac{1}{k!}\begin{pmatrix} \mu_1 & \cdots & \mu_k \\ a_1 & \cdots & \alpha_k \end{pmatrix}\begin{pmatrix} \nu_1 & \cdots & \nu_k \\ a_1 & \cdots & \alpha_k \end{pmatrix}),$$

completely define the coefficients at the principal derivatives. For example, the simplest compatible equation (non-Lagrangian; see Einstein–Mayer classification of compatible equations in 4D AP [3]), $\mathbf{E}^*{}_{a\mu} = \Lambda_{a\mu\nu;\nu} = 0$, with evident identity $\mathbf{E}^*{}_{a\mu;\mu} \equiv 0$ (required for formal integrability) can be written as

$$h^2 \mathbf{E}^*{}_a{}^\mu = (h_{a\alpha,\beta\nu} - (\alpha\beta))(-g) g^{\alpha\mu} g^{\beta\nu} + (h'^2) = h_{a\alpha,\beta\nu}[\alpha\mu, \beta\nu] + (h'^2)\ . \tag{4}$$

Like determinant, the k-minors ($k \leq D$) are multi-linear expressions with respect to the elements of $h^a{}_\mu$ matrix, and some minors do not vanish when $\mathrm{rank} h^a{}_\mu = D - 1$.

It turns out that for any AP equation [including Eqs. (3) and (4)], with the unique exception, Eq. (1), where only skew-symmetric part participates in identity (and can be written with 2- and 3-minors, while symmetric part requires 1-minors which vanish too rapidly), regularity of principal terms survives (and symbol G_2 keeps involutive [1]) when $\mathrm{rank} g_{\mu\nu} = D - 1$.

This observation is important and relevant to the problem of singularities of solutions; it means seemingly that the unique equation (1) does not suffer of nascent co-singularities in solutions of general position.

The other case is contra-singularities [2] relating to degeneration of contra-variant density of some weight:

$$H_a{}^\mu = h^{1/D_*} h_a{}^\mu ; H = \det H^a{}_\mu,\ \ h_a{}^\mu = H^{1/(D-D_*)} H_a{}^\mu\ . \tag{5}$$

Here D_* depends on equation: $D_* = 2$ for GR, $D_* = \infty$ for Eq. (4), and $D_* = 4$ for the unique equation (which can be written in 3-linear form with respect to $H_a{}^\mu$ and its derivatives [2]).

If integer, D_* has the meaning of forbidden space dimension. The nearest possible D for the unique equation, $D = 5$, is of special interest because in this case the minor $H^{-1} H^a{}_\mu$ just coincides with $h^a{}_\mu$; that is, contra-singularity simultaneously implies co-singularity (of very high co-rank), but that is impossible! The possible interpretation of this observation is that (perhaps due to some specific of *Diff*-orbits on $H_a{}^\mu$-space at contra-singularity), for the unique equation, contra-singularities (of solutions of general position) are impossible if $D = 5$. This leaves no room for changes in the theory (if to admit that the Nature does not like singularities).

2 Tensor $T_{\mu\nu}$ (despite Lagrangian absence) and PN-effects (Pauli's questions to AP)

One might rearrange $\mathbf{E}_{(\mu\nu)} = 0$ that to pick out (say into LHS) the Einstein tensor, $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$, but the rest terms are not proper energy-momentum tensor: they contain linear terms $\Phi_{(\mu;\nu)}$ (no positive energy (!); instead one more presentation of 'Maxwell equation' (2) is possible – as divergence of symmetrical tensor).

However, the prolonged equation $\mathbf{E}_{(\mu\nu);\lambda;\lambda} = 0$ can be written as 'plain' (no R-term) RG-gravity:

$$(-h^{-1}\,\delta(h\,R_{\mu\nu}G^{\mu\nu})/\delta g_{\mu\nu}{=})\; G_{\mu\nu;\lambda;\lambda} + G_{\epsilon\tau}(2R_{\epsilon\mu\tau\nu} - \tfrac{1}{2}g_{\mu\nu}R_{\epsilon\tau}) = T_{\mu\nu}(\Lambda'^2, \cdots),\;\; T_{\mu\nu;\nu} = 0; \tag{6}$$

up to quadratic terms, $T_{\mu\nu} = \frac{2}{9}(\frac{1}{4}g_{\mu\nu}f^2 - f_{\mu\lambda}f_{\nu\lambda}) + A_{\mu\epsilon\nu\tau}(\Lambda^2)_{,\epsilon\tau}$;

tensor A has symmetries of Riemann tensor, so the term A'' adds nothing to momentum and angular momentum.

It is worth noting that:

(a) the theory does not match GR, but shows 'plain' R^2-gravity (sure, (6) does not contain all the theory);

(b) only f-component (three transverse polarizations in D=5) carries D-momentum and angular momentum ('powerful' waves); other 12 polarizations are 'powerless', or 'weightless' (this is a very unusual feature – impossible in the Lagrangian tradition; how to quantize ? let us not to try this leaving the theory 'as is');

(c) f-component feels only metric and S-field ('contorsion', not 'torsion' Λ – to label somehow), see (2), but S has effect only on polarization of f: $S_{[\mu\nu\lambda]}$ does not enter eikonal equation, and f moves along usual Riemannian geodesic (if background has f=0); one may think that all 'quantum fields' (phenomenological quantized fields accounting for topological (quasi)charges and carrying some 'power'; see further) inherit this property;

(d) the trace $T_{\mu\mu} = \frac{1}{18}f_{\mu\nu}f_{\mu\nu}$ can be non-zero if $f^2 \neq 0$ and this seemingly depends on S-component (which enters the current in (2)); in other words, 'mass distribution' is to depend on distribution of f- and S-component;

(e) it should be stressed and underlined that the f-component is not usual (quantum) EM-field – just important covariant responsible for energy-momentum (suffice it to say that there is no gradient invariance for f).

3 Linear domain: stable and unstable (powerless) waves. Instability of trivial solution. Expanding O_4-symmetrical solutions and cosmology

Another strange feature is the instability of trivial solution: some 'powerless' polarizations grow linearly with time in presence of 'powerful' f-polarizations. Really, from the linearized Eq. (1) (and the identity below it) one can write (the following equations should be understood as linearized):

$$\Phi_{a,a} = 0 \ (D \neq 4), \ \ 3\Lambda_{abd,d} = \Phi_{a,b} - 2\Phi_{b,a}, \ \ \Lambda_{a[bc,d],d} \equiv 0 \ \ \Rightarrow \ \ 3\Lambda_{abc,dd} = -2f_{bc,a} \, .$$

The last 'D'Alembert equation' has the 'source' in its right hand side. Some components of Λ (most symmetrical irreducible parts) do not grow (as well as curvature), because (again, linearized equations are implied below)

$$S_{abc,dd} = 0, \ \ \Phi_{a,dd} = 0, \ \ f_{ab,dd} = 0, \ \ R_{abcd,ee} = 0,$$

but the least symmetrical components of the tensor Λ do go up with time (due to terms $\sim t\, e^{-i\omega t}$; three growing polarizations which are 'imponderable', or powerless) if the 'ponderable' waves (three f-polarizations) do not vanish (and this should be the case for solutions of 'general position').

The unique symmetry of AP equations gives scope for symmetrical solutions. In contrast to GR, the Unique Eqn has non-stationary spherically symmetric solutions. The O_4-symmetric field can be generally written [2] as

$$h^a{}_\mu(t, x^i) = \begin{pmatrix} a & bn_i \\ cn_i & en_in_j + d\Delta_{ij} \end{pmatrix}; \quad i, j = (1, 2, 3, 4), \ \ n_i = \frac{x^i}{r}. \tag{7}$$

Here $a, \ldots, e$ are functions of time, $t = x^0$, and radius r, $\Delta_{ij} = \delta_{ij} - n_in_j$, $r^2 = x^ix^i$. As functions of radius, b, c are odd, while the others are even; other boundary conditions: $e = d$ at $r = 0$, and $h^a{}_\mu \to \delta^a_\mu$ as $r \to \infty$. Placing in (7) $b = 0, e = d$ (the other interesting choice is b=c=0) and making integrations one can arrive to the next system (resembling dynamics of Chaplygin gas; dot and prime denote derivation on time and radius, resp.)

$$A^{\cdot} = AB' - BA' + 3AB/r \ , \ \ B^{\cdot} = AA' - BB' - 2B^2/r \ , \ \text{where } A = a/e = e^{1/2}, \ \ B = -c/e. \tag{8}$$

This system has non-stationary solutions, and a single-wave solution (of proper 'amplitude') might serve as a suitable cosmological (expanding) background. The condition $f_{\mu\nu}=0$ is a must for solutions with such a high symmetry (as well as $S_{\mu\nu\lambda}=0$); so, these O_4-solutions carry no energy, that is, weight nothing (some lack of *gravity* ! in this theory the universe expansion seemingly has little common with GR and its dark energy [4]).

More realistic cosmological model might look like a single O_4-wave (or a sequence of such waves) moving along the radius and being filled with chaos, or stochastic waves, both powerful (*weak*, $\Delta h \ll 1$) and powerless ($\Delta h < 1$, but intense enough that to lead to non-linear fluctuations with $\Delta h \sim 1$), which form statistical ensemble(s) having a few characteristic parameters (after 'thermalization'). The development and examination of stability of such a model is an interesting problem. The inhomogeneity of metric in the cosmological O_4-wave can serve as a time-dependent 'shallow dielectric guide' for that weak noise waves. The ponderable waves (which slightly 'decelerate' the O_4-wave) should have wave-vectors almost tangent to the S^3-sphere of wave-front that to be trapped inside this ('shallow') wave-guide; the imponderable waves can grow up, and partly escape from the wave-guide, and their wave-vectors can be less tangent to the S^3-sphere.

The waveguide thickness can be small for an observer in the center of O_4-symmetry, but in co-moving coordinates it can be very large (relativistic effect), however still small with respect to the radius of sphere, $L \ll R$. It seems that the radial dimension has to be very 'undeveloped': there are no other characteristic scales, $< L$.

4 Non-linear domain: topological charges and quasi-charges

Let AP-space is of trivial topology: no worm-holes, no compactified space dimensions, no singularities. One can continuously deform frame field $h(x)$ to a field of rotation matrices (metric can be diagonalized and 'square-rooted') $h^a{}_\mu(x) \rightarrow s^a{}_\mu(x) \in SO(1,d)$; $m{=}D{-}1$. Further deformation can remove boosts too, and so, for any space-like (Cauchy) surface, this gives the (pointed) map, $s\,:\,\mathbf{R}^m \cup \infty = S^m \rightarrow SO_m$; $\infty \mapsto 1^m \in SO_m$.

The set of such maps consists of homotopy classes which form the group of topological charge, $\Pi(m)$:

$$\Pi(m) = \pi_m(SO_m); \quad \Pi(3) = Z, \;\Pi(4) = Z_2 + Z_2. \tag{9}$$

Here Z is the infinite cyclic group, and Z_2 is the cyclic group of order two.

It is important that deformation to s-field can keep symmetry of field configuration. Definition: localized field (pointed map) $s(x): \ \mathbf{R}^m \to SO(m), \ s(\infty) = 1^m$, is G-symmetric if, in some coordinates,

$$s(\sigma x) = \sigma s(x) \sigma^{-1} \ \ \forall\, \sigma \in G \subset O(m)\,. \tag{10}$$

The set of such fields $\mathcal{C}_G^{(m)}$ generally consists of separate, disconnected components – homotopy classes forming the 'topological quasi-charge group' denoted here as $\Pi(G; m) \equiv \pi_0(\mathcal{C}_G^{(m)})$. These QC-groups classify symmetrical localized configurations of frame field. Since field equation does not break symmetry, quasi-charge conserves; if symmetry is not exact (because of distant regions), quasi-charge is not exactly conserving value, and quasi-particle (of zero topological charge) can vanish or arise during colliding with another quasi-particle.

The other problem. Let $G1 \supset G2$, such that there is a mapping (embedding) $i: \ \mathcal{C}_{G1}^{(m)} \to \mathcal{C}_{G2}^{(m)}$, which induces the homomorphism of QC-groups: $i_*: \ \Pi(G1; m) \to \Pi(G2; m)$, so one has to describe this morphism.

Let us consider the very simple (discreet) symmetry group P_1 with a plane of reflection symmetry:

$$P_1 = \{1, p_{(1)}\}, \quad \text{where } p_{(1)} = \mathrm{diag}(-1, 1, \ldots, 1) = p_{(1)}^{-1}.$$

It is necessary to set field $s(x)$ on the half-space $\frac{1}{2}\mathbf{R}^m = \{x^1 \geq 0\}$, with additional condition imposed on the surface $\mathbf{R}^{m-1} = \{x_1 = 0\}$ (stationary points of P_1 group) where s has to commute with the symmetry [see (10)]:

$$p_{(1)} x = x \ \Rightarrow \ s(x) = p_{(1)} s p_{(1)} \ \Rightarrow \ s \in 1 \times SO_{m-1}.$$

Hence, accounting for the localization requirement, we have a diad map (relative spheroid; here D^m is an m-ball and S^{m-1} its surface) $(D^m; S^{m-1}) \to (SO_m; SO_{m-1})$, and topological classification of such maps leads to the relative (or diad) homotopy group ([5]; the last equality below follows due to fibration $SO_m/SO_{m-1} = S^{m-1}$):

$$\Pi(P_1; m) = \pi_m(SO_m; SO_{m-1}) = \pi_m(S^{m-1}).$$

Similar considerations (of group orbits and stationary points) lead to the following result:

$$\Pi(O_l; m) = \pi_{m-l+1}(SO_{m-l+1}; SO_{m-l}) = \pi_{m-l+1}(S^{m-l}).$$

If $l > 3$, there is the equality: $\Pi(SO_l; m) = \Pi(O_l; m)$, while for $l = 2, 3$ one can find:

$$\Pi(SO_3; m) = \pi_{m-2}(SO_2 \times SO_{m-2}; SO_{m-3}) = \pi_{m-2}(S^1 \times S^{m-3}),$$

$$\Pi(SO_2; m) = \pi_{m-1}(SO_m; SO_{m-2} \times SO_2) = \pi_{m-1}(RG_+(m, 2)).$$

The set of quaternions with absolute value one, $\mathbf{H}_1 = \{\mathbf{f},\ |\mathbf{f}| = 1\}$, forms a group under quaternion multiplication, $\mathbf{H}_1 \cong SU_2 = S^3$, and any $s \in SO_4$ can be represented as a pair of such quaternions [5], $(\mathbf{f},\ \mathbf{g}) \in S^3_{(l)} \times S^3_{(r)}$, $|\mathbf{f}| = |\mathbf{g}| = 1$: $x^* = sx \;\Leftrightarrow\; \mathbf{x}^* = \mathbf{f}\,\mathbf{x}\,\mathbf{g}^{-1} = \mathbf{f}\,\mathbf{x}\,\bar{\mathbf{g}}\,; \quad |\mathbf{x}| = |\mathbf{x}^*|.$

The pairs $(\mathbf{f}, \mathbf{g})$ and $(-\mathbf{f}, -\mathbf{g})$ correspond to the same rotation s, that is, $SO_4 = S^3_{(l)} \times S^3_{(r)}/\pm$. Note that the symmetry condition (10) also splits into two parts:

$$\mathbf{f}(\mathbf{a}\mathbf{x}\mathbf{b}^{-1}) = \mathbf{a}\mathbf{f}(\mathbf{x})\mathbf{a}^{-1},\ \mathbf{g}(\mathbf{a}\mathbf{x}\mathbf{b}^{-1}) = \mathbf{b}\mathbf{g}(\mathbf{x})\mathbf{b}^{-1}\ \forall(\mathbf{a}, \mathbf{b}) \in G \subset SO_4. \tag{11}$$

Example of SO_2-symmetric quaternion field. Let's consider an example of $SO_2\{2, 3\}-$symmetric $\mathbf{f}$–field configuration ($\mathbf{g}$=1), which carries both charge and SO_2-quasi-charge (left, of course), $\mathbf{f}(\mathbf{x})$: $\mathbf{H} = \mathbf{R}^4 \to \mathbf{H}_1$; $\mathbf{f}(\infty) = 1$. The symmetry condition (11) reads

$$\mathbf{f}(e^{\mathbf{i}\,\phi/2}\mathbf{x}e^{-\mathbf{i}\,\phi/2}) = e^{\mathbf{i}\,\phi/2}\mathbf{f}(\mathbf{x})e^{-\mathbf{i}\,\phi/2}. \tag{12}$$

We'll switch to 'double-axial' coordinates: $\mathbf{x} = ae^{\mathbf{i}\,\varphi} + be^{\mathbf{i}\,\psi}\mathbf{j}$. Let us use imaginary quaternions $\mathbf{q}$ as stereogrphic coordinates on $\mathbf{H}_1$, and take symmetrical field $\mathbf{q}(\mathbf{x})$ consistent with Eq. (12):

$$\mathbf{q}(\mathbf{x}) = \mathbf{x}\,\mathbf{i}\,\bar{\mathbf{x}} + \mathbf{i} = -\bar{\mathbf{q}},\ \ \mathbf{f}(\mathbf{x}) = -\frac{1+\mathbf{q}}{1-\mathbf{q}} = 1 - \frac{2}{1-\mathbf{q}}. \tag{13}$$

It is easy to find the 'center of quasi-soliton' (1-submanifold, S^1)

$$S^1 = \mathbf{f}^{-1}(-1) = \mathbf{q}^{-1}(0) = \{a = 0,\ b = 1\} = \{\mathbf{x}_0(\psi) = e^{\mathbf{i}\,\psi}\mathbf{j}\}$$

and the 'vector equipment' on this circle: $d\mathbf{x}|_{\mathbf{x}_0} = da\, e^{\mathbf{i}\,\varphi} + (db + \mathbf{i}\, d\psi)e^{\mathbf{i}\,\psi}\mathbf{j}$, $\frac{1}{4}d\mathbf{f}\big|_{\mathbf{x}_0} = \mathbf{i}db - \mathbf{k}\, e^{\mathbf{i}\,(\varphi+\psi)}da$; $\mathbf{i}$-vector all time looks along the radius b (parallel translation along the circle S^1; this is a 'trivial', or 'flavor'-vector). Two others ('phase'-vectors) make $2\pi-$rotation along the circle.

In fact, the field (13) has also symmetry $SO_2\{1,4\}$, and this feature restricts possible directions of 'flavor'-vector (two 'flavors' are possible, $\pm$; the $P_2\{1,4\}-$symmetry (this is the π-rotation of x^1, x^4) gives the same effect). The other interesting observation is that the equipped circle can be located also at the stationary points of SO_2-symmetry (this increases the number of 'flavors').

5 Quasi-charges and their morphisms (in $5D$, ie $m = 4$)

If $G \subset SO_4$, the QC-group has two isomorphous parts, left and right: $\Pi(G) = \Pi_{(l)}(G) + \Pi_{(r)}(G)$. The Table below describes quasi-charge groups for $G \subset G_0 = (O_3 \times P_4) \cap SO_4$ (P_4 is spatial inversion, the 4-th coordinate is the extra dimension (along the radius) of G_0-symmetric expanding cosmological background).[1]

One can assume further that an hadron bag is a specific place where G_0-symmetry does not work, and the bag's symmetry is isomorphous to O_4. This assumption can lead to another classification of quasi-solitons (some

[1] *Quasi-particles*, which symmetry includes P_4, seem to be true neutral (neutrinos, photon, Higgs particles; the last should have spin [and perhaps flavour] – SO_3-symmetrical [i.e. spin zero] *elementary particles* should not exist).

doubling the above scheme), where self-dual and anti-self-dual one-parameter groups take place of SO_2-group. The total set of quasi-particle parameters (parameters of equipped 1-manifold (loop) plus parameters of group) for (anti)self-dual groups, $G(4,2) \times RP^2$, is larger than the analogous set for groups $SO_2 \subset G_0$, which is just $O_3 \times G(3,1) = RP^2$. If the number of 'flavor'-parameters (which are not degenerate and have some preferable particular values; this should be sensitive to discreet part of G – at least photons have the same flavor) is the same as in the case of 'white' quasi-particles, the remaining parameters (degenerate, or 'phase') can give room for 'color' (in addition to spin). So, perhaps one might think about 'color neutrinos' (in the context of pomeron, and baryon spin puzzle), 'color W, Z, and Higgs' (another context – B-mesons), and so on.

Table. QC-groups $\Pi_{(l)}(G)$ and their morphisms to the preceding group; $G \subset G_0$.

G	$\Pi_l(G) \to \Pi_l(G^*)$	'label'
1	Z_2	
$SO\{1,2\}$	$Z_{(e)} \xrightarrow{e} Z_2$	e
$SO\{1,2\} \times P\{3,4\}$	$Z_{(\nu)} + Z_{(H)} \xrightarrow{i,m2} Z_{(e)}$	$\nu^0;\ H^0 \to e+e$
$SO\{1,2\} \times P\{2,3\}$	$Z_{(W)} \xrightarrow{0} Z_{(e)}$	$W \to e + \nu^0$
$SO\{1,2\} \times P\{2,4\}$	$Z_{(Z)} \xrightarrow{0} Z_{(e)}$	$Z^0 \to e + e$
$SO\{1,2\} \times P\{3,4\} \times$ $\times P\{2,3\}$	$Z_{(\gamma)} \xrightarrow{0} Z_{(H)}$ $\xrightarrow{0} Z_{(W)}$	$\gamma^0 \to H^0 + H^0$ $\to W + W$

Note that in this picture the very notion of quasi-particle depends on the background symmetry (also to note: there are no 'quanta of torsion' per se). On the other hand, large clusters of quasi-particles (matter) can disturb the background, and waves of such small disturbances (with wavelength larger than the thickness L, perhaps) can be generated as well (but these waves do carry no (quasi)charges, that is, are *not quantized*).

Coexistence: phenomenological 'quantum fields' on classical background. The non-linear, particle-like field configurations with quasi-charges (quasi-particles) should be very elongated along the extra-dimension (all of the same size L), while being small sized along usual dimensions, $\lambda \ll L$. The motion of such a spaghetti-like quasi-particle should be very complicated and stochastic due to 'strong' imponderable noise, such that different parts of spaghetti are coming their own paths. At the same time, quasi-particle can acquire 'its own' energy–momentum – due to scattering of ponderable waves (which wave-vectors are almost tangent to usual $3D$ (sub)space); so, it seems that scattering amplitudes[2] of those spaghetti's parts which have the same $3D$–coordinates can be summarized providing an auxiliary, secondary field.

So, the imponderable waves provides stochasticity (of motion of spaghetti's parts), while the ponderable waves ensure superposition (with secondary fields). Phenomenology of secondary fields could be of Lagrangian type, with positive energy acquired by quasi-particles, – that to ensure the stability (of all the waveguide with its infill – with respect to quasi-particle production; the least action principle has deep concerns with Lyapunov stability and is deducible, in principle, from the path integral approach).

6 'Plain' R^2 gravity on very thick brane and change in the Newton's Gravity Law

Let us start with 4d (from 5D) bi-Laplace equation with a δ-source [as weak field, non-relativistic (stationary) approximation (it is assumed that 'mass is possible') from R^2 gravity (6)] and its solution:

$$\Delta^2\varphi = -\frac{a}{R^3}\,\delta(R);\;\; \varphi(R^2) = \frac{a}{8}\ln R^2 - \frac{b}{R^2}\;(+\,c\,,\ \text{but } c \text{ does not matter; } R \text{ is 4d distance}); \tag{14}$$

the attracting force between two point masses is $F_{\text{point}} = \frac{a}{4R} + \frac{2b}{R^3}$, a, b should be proportional to both masses.

Now let us suppose that all masses are distributed along the extra dimension with a 'universal function',

[2] These amplitudes can depend on additional vector-parameters ('equipment vectors') relating to differential of field mapping at a 'quasi-particle center' – where quasi-charge density is largest (if it has covariant sense).

$\mu(p)$, $\int\mu(p)\,dp = 1$. Then the attracting (gravitation) force takes the next form [see (14); r is usual 3d distance]:

$$F(r) = \frac{d}{dr}\iint_{-\infty}^{\infty}\varphi(r^2+(p-q)^2)\,\mu(p)\,\mu(q)\,dp\,dq = \frac{a\,r}{4}V(r) - b\,V', \;\; V(r) = \iint\frac{\mu(p)\,\mu(q)}{r^2+(p-q)^2}\,dp\,dq. \tag{15}$$

(Note that $V(r)$ can be restored if $F(r)$ is measured.) Taking $\mu_1(p) = \pi^{-1}/(1+p^2)$ (typical scale along extra dimensions is taken as unit, $L = 1$; it seems that L should be greater than ten AU), one can find

$$F(r) = \frac{a}{8+4r} + \frac{2b(1+r)}{r^2(2+r)^2}; \;\text{ or (now } L\neq 1)\; F(r) = \frac{1}{r^2} + \frac{r}{2L(2L+r)^2}, \;\; \text{where } a = b = 2/L^2.$$

Fig. 1, curve (a) shows $\delta F = F - 1/r^2$ (deviation from the Newton's Law of Gravity; a/b is chosen that $\delta F(0){=}0$); two other curves, (b) & (c), correspond to $\mu_2 = 2\pi^{-1}/(1+p^2)^2$, $\mu_3 = 2\pi^{-1}p^2/(1+p^2)^2$ ($\delta F(0){=}0$). We see that in principle this theory can explain galaxy rotation curves, $v^2(r) \propto rF \overset{r\to\infty}{\longrightarrow}$ const, without need for Dark Matter (or MOND; about rotation curves and DM see [6]; they are looking for DM in Solar system too, [7]).

Q: Can the 'coherence of mass' along the extra dimension be disturbed? (flyby and Pioneer anomalies [8])

7 Single photon experiment (that to feel very large extra dimension), and conclusion

Today, many laboratories have sources of single (heralded) photons, or entangled bi-photons (say, for Bell-type experiments [9]); some students can perform laboratory works with single photons, having convinced on their own experience that light is quantized (the Grangier experiment)[10].

It is being suggested a minor modification of the single (polarized) photon interference experiment, say, in a Mach-Zehnder fiber interferometer with 'long' (the fibers may be rolled) enough arms. The only new element is a fast-acting shutter placed at the beginning of (one of) the interferometer's arms (the closing-opening time of the shutter should be smaller than the flight time in the arms). For example, a fast electro-optical modulator in combination with polarizer (or a number of such combinations) can be used with polarized photons.

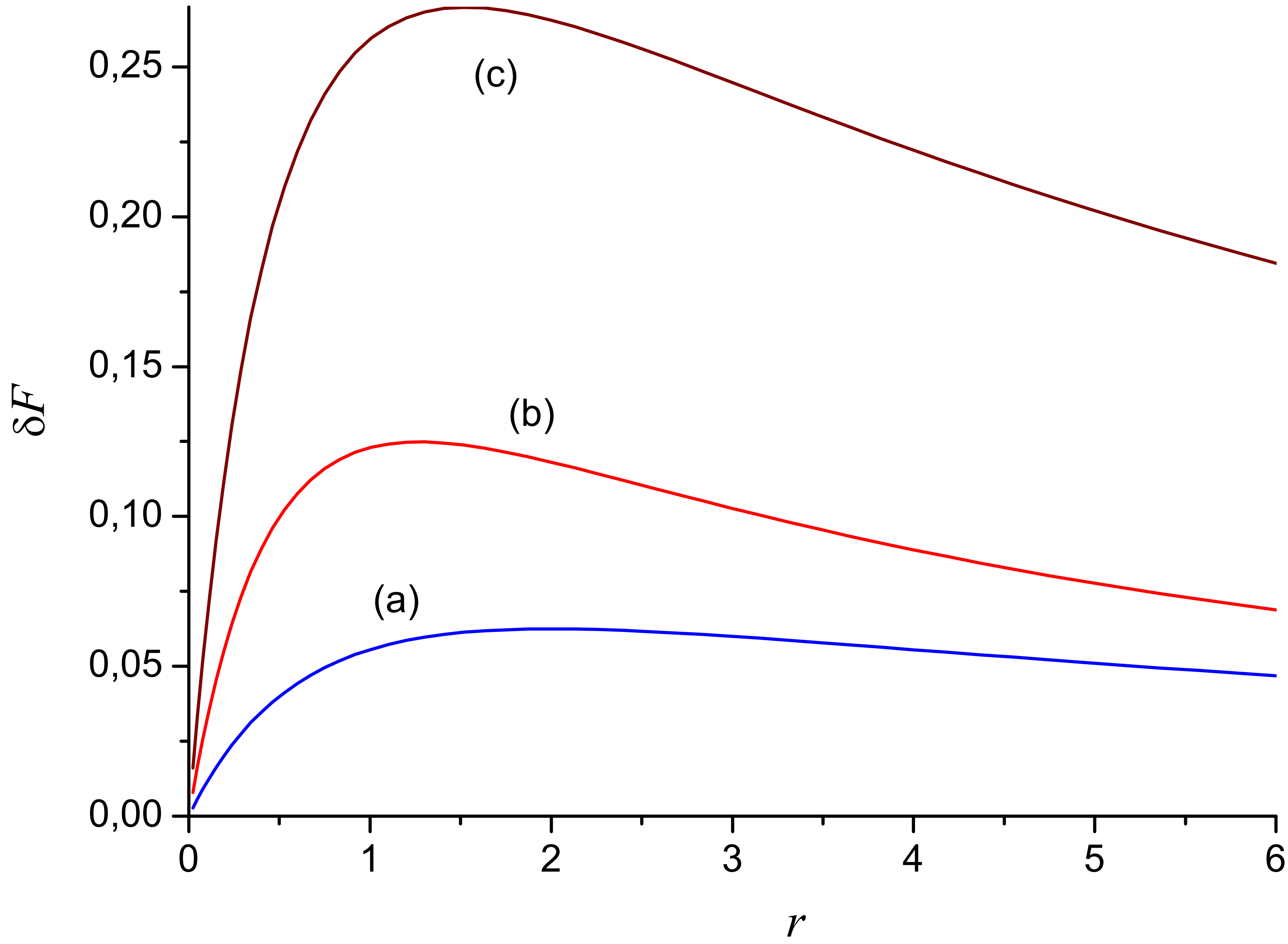

Fig. 1. Deviation $\delta F = F - 1/r^2$ for different $\mu(p)$, see the Eq. (15) and text below.

Both Quantum mechanics (no particle's ontology) and Bohmian mechanics (wave-particle double ontology)exclude any change in the interference figure as a result of separating activity of such a fast shutter (while the photon's 'halves' are making their ways to the place of a meeting). However, if a photon has non-local spaghetti-like ontology (along the extra dimension) and fragments of this spaghetti are moving along both arms at once, then the shutter should tear up this spaghetti (mainly without photon absorption), tear out its fragments (which will dissolve in 'zero-point oscillations'). Hence, if the absorption factor of the shutter (the extinction ratio of polarizer) is large enough, the 50/50-proportion (between the photon's amplitudes in the arms) will be changed and *a significant decrease of the interference visibility should be observed*.

QM is everywhere (where we can see, of course), and, so, non-linear $5D$-field fluctuations, looking like spaghetti-anti-spaghetti loops, should exist everywhere. (This omnipresence can be related to the universality of 'low-level heat death', restricted by the presence of topological quasi-solitons – some as the $2D$ computer experiment by Fermi, Pasta, and Ulam, where the process of thermalization was restricted by the existence of solitons. See also the above sections (and [2]) for arguments in favor of phenomenological (quantized) 'secondary fields' accounting for topological (quasi)charges and obeying superposition, path integral and so on.)

AP, at least at the level of its symmetry, seems to be able to cure the gap between the two branches of physics – General Relativity (with coordinate diffeomorphisms) and Quantum Mechanics (with Lorentz invariance). Most people give all the rights of fundamentality to quanta, and so, they are trying to quantize gravity, and the very space-time (probing loops, and strings, and branes; see also the warning polemic by Schroer [11]). The other possibility is that quanta have the specific phenomenological origin relating to topological (quasi)charges.

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*The other popular sources of contra-string polemics are seemingly the following books: P. Woit, *Not even wrong*; L. Smolin, *The Trouble with Physics* (as well as the blog www.math.columbia.edu/∼woit/wordpress/).

**MOND link: www.astro.umd.edu/∼ssm/mond/