

On relevance of a modified gravity

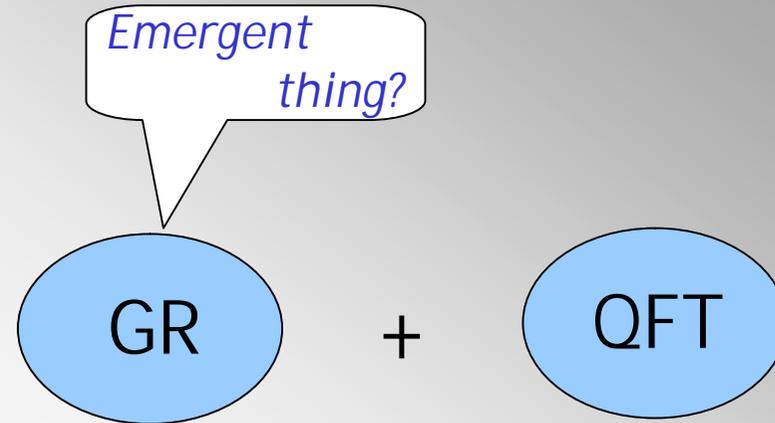
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Two main theories:

General Relativity (general covariance)
Quantum Field Theory, SM (Lorentz covariance, ...)

Vacuum GR is a single field theory (irreducibility);
but – the problem of singularities;
free parameter, 'constant'

Different interpretation of QM:
Copenhagen and Many-World
(of similar 'rating')
Both interpretations are equally convincing



Lagrangian approach is their common denominator

Something else:
single field theory?

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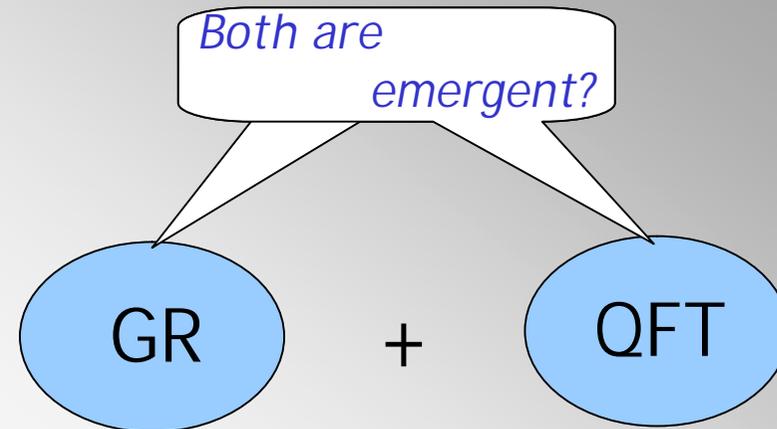
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Quantum Field Theory, SM (Lorentz covariance, ...)

Vacuum GR is a single field theory (irreducibility);
but – the problem of singularities;
free parameter, ‘constant’

Different interpretation of QM:
Copenhagen and Many-World
(of similar ‘rating’)
Both interpretations are equally unconvincing

So, is it possible that
both are emergent ?

Absolute Parallelism:
embraces symmetries of both GR and SR
(plus irreducibility); topological charges and quasi-charges;
instability of the trivial solution; unique variant with $D=5$ -- no singularities;
the emergent phenomenology of topological quanta can look like a QFT + mod.gravity



Lagrangian approach is their common denominator

Something else:
single field theory?

Absolute Parallelism (frame field theory)

Riemann-squared gravities

($a+bR+R^2/2$)–gravity (4th order equation):

$$\mathbf{E}_{\mu\nu} = R_{;\mu;\nu} - R_{\mu\nu}(b + R) + g_{\mu\nu}(\dots R..R^2) = 0$$

new 3^d order eq-ns (irregular, in the second jets):

$$0 = \mathbf{E}_{\mu\nu;\lambda} - \mathbf{E}_{\mu\lambda;\nu} = \underline{R_{;\varepsilon} R^{\varepsilon}_{\mu\nu\lambda}} + (R_{\mu\nu}, R)$$

The same is valid for other $F(R)$ –gravities (if $F(R) \neq R$).

Eq-ns of Gauss-Bonnet (Lovelock) gravity are irregular too (in second jets).

Ricci-squared gravities: instability of the trivial solution

RG-gravity (Ricci-Einstein),

$$L = R_{mn} G^{mn} : \quad -D_{\mu\nu} = G_{\mu\nu;\lambda}{}^{;\lambda} + G^{\epsilon\tau} (2R_{\epsilon\mu\tau\nu} - \frac{1}{2}g_{\mu\nu}R_{\epsilon\tau}) = 0; \quad D_{\mu\nu;\lambda}g^{\nu\lambda} \equiv 0.$$

$$\mathcal{L} = \sqrt{-g}L, \quad \mathcal{L}(\kappa g_{\mu\nu}) = \kappa^p \mathcal{L}(g_{\mu\nu}) \quad \sqrt{-g}D^{\mu\nu}g_{\mu\nu} = \frac{\delta\mathcal{L}}{\delta g_{\mu\nu}}g_{\mu\nu} = p\mathcal{L} + (\sqrt{-g}A^\mu)_{,\mu}$$

Bianchi identity,
prolongation & contractions

$$R_{\mu\nu[\lambda\epsilon;\tau]} \equiv 0 \quad R_{\mu\nu[\lambda\epsilon;\tau];\rho}g^{\tau\rho} \equiv 0, \quad R_{\mu\nu[\lambda\epsilon;\tau]}g^{\mu\tau} \equiv 0$$

Linear approximation,
and linear instability
(related to Weyl tensor):
Ricci tensor is a 'source'
of this instability

$$\square R = 0, \quad \square R_{\mu\nu} = 0$$

$$\square R_{\mu\epsilon\nu\tau} = R_{\mu\nu,\epsilon\tau} - R_{\mu\tau,\nu\epsilon} + R_{\epsilon\tau,\mu\nu} - R_{\epsilon\nu,\mu\tau}$$

$$a(t) = (c_0 + c_1 t) \exp(-i\omega t)$$

Generally (e.g. Ricci-squared gravity,
 $L = R_{mn}R^{mn}$), more strong instability,
quadratic growth of Weyl tensor:

$$\square R = 0, \quad \square R_{\mu\nu} \propto R_{,\mu\nu} \quad R_{\mu\epsilon\nu\tau} \propto t^2$$

$$\square R_{\mu\epsilon\nu\tau} = R_{\mu\nu,\epsilon\tau} - R_{\mu\tau,\nu\epsilon} + R_{\epsilon\tau,\mu\nu} - R_{\epsilon\nu,\mu\tau}$$

Weak gravity is impossible here !!

AP

Interesting features of Absolute Parallelism

- High symmetry of equation
+ irreducibility of frame field :
includes symmetries of both SR and GR; metric is a quadratic form

$$\tilde{h}^a{}_{\mu}(\tilde{x}) = \kappa \sigma^a{}_b h^b{}_{\nu}(x) \frac{\partial x^{\nu}}{\partial \tilde{x}^{\mu}}$$

$$\kappa > 0, \sigma^a{}_b \in O(1, D-1)$$

$$g_{\mu\nu} = \eta_{ab} h^a{}_{\mu} h^b{}_{\nu}, \quad \eta_{ab}, \eta^{ab} = \text{diag}(-1, 1, \dots, 1)$$

- Absence of arising singularities and uniqueness:

there is one unique variant of AP, with the unique D, D=5, which solutions are free of singularities. No room for changes !

First order covariant:

$$\Lambda_{a\mu\nu} = h_{a\mu,\nu} - h_{a\nu,\mu} = 2h_{a[\mu;\nu]} \quad (g_{\mu\nu;\lambda} \equiv 0)$$

$$S_{abc} = 3\Lambda_{[abc]} = \Lambda_{abc} + (abc), \quad \Phi_a = \Lambda_{b\mu\nu} h_b{}^{\mu} h_a{}^{\nu}$$

$$R_{a\mu\nu\lambda} = 2h_{a\mu;[\nu;\lambda]}; \quad h_{a\mu} h_{a\nu;\lambda} = \frac{1}{2} S_{\mu\nu\lambda} - \Lambda_{\lambda\mu\nu}$$

- Topological features:

field configurations with topological charge and/or quasi-charge (topological quanta)

$$\mathbf{E}_{a\mu} = L_{a\mu\nu;\nu} - \frac{1}{3}(f_{a\mu} + L_{a\mu\nu}\Phi_{\nu}) = 0$$

$$L_{a\mu\nu} = L_{a[\mu\nu]} = \Lambda_{a\mu\nu} - S_{a\mu\nu} - \frac{2}{3}h_{a[\mu}\Phi_{\nu]}$$

- Energy-momentum tensor (positive energy):

conservation laws arise in presence of symmetries (Killing vectors), or in weak field; but most 'polarizations' do not contribute to energy (powerless, intangible waves)

- Instability of trivial solution (growing intangible waves); non-stationary O_4 -symmetric solution (single wave) as more appropriate expanding cosmological background – to be filled with stochastic waves and topological quanta

Linear instability of the trivial solution

$$\square R_{\mu\epsilon\nu\tau} = R_{\mu\nu,\epsilon\tau} - R_{\mu\tau,\nu\epsilon} + R_{\epsilon\tau,\mu\nu} - R_{\epsilon\nu,\mu\tau} \quad R_{\mu\nu} \propto \Phi_{(\mu,\nu)}$$

First order covariant
(and its irreducible parts):

$$\Lambda_{a\mu\nu} = h_{a\mu,\nu} - h_{a\nu,\mu} = 2h_{a[\mu;\nu]} \quad (g_{\mu\nu;\lambda} \equiv 0)$$

$$S_{abc} = 3\Lambda_{[abc]} = \Lambda_{abc} + (abc), \quad \Phi_a = \Lambda_{b\mu\nu} h_b^\mu h_a^\nu$$

Tensor f_{mn} (only it carries energy-momentum).

Identities:

$$f_{\mu\nu} = 2\Phi_{[\mu;\nu]} = \Phi_{\mu;\nu} - \Phi_{\nu;\mu}; \quad f_{[\mu\nu;\lambda]} \equiv 0$$

$$\Lambda_{a[\mu\nu;\lambda]} \equiv 0 \quad \Lambda_{abc,a} + f_{bc} \equiv 0 \quad (,a = ;_\mu h_a^\mu)$$

Field eq-ns:

$$\mathbf{E}_{a\mu} = L_{a\mu\nu;\nu} - \frac{1}{3}(f_{a\mu} + L_{a\mu\nu}\Phi_\nu) = 0$$

$$L_{a\mu\nu} = L_{a[\mu\nu]} = \Lambda_{a\mu\nu} - S_{a\mu\nu} - \frac{2}{3}h_{a[\mu}\Phi_{\nu]}$$

$$\mathbf{E}_{(\mu\mu)} = \frac{4-D}{3}\Phi_{\mu;\mu} + (\Lambda^2)$$

$$D \neq 4$$

Linear approximation,
and linear instability:

unstable, ie growing polarizations, $\sim t$

$$3\Lambda_{abd,d} = \Phi_{a,b} - 2\Phi_{b,a} \quad \Phi_{a,a} = 0$$

$$\square\Phi_a = 0$$

$$\Lambda_{a[bc,d],d} \equiv 0 \Rightarrow \square\Lambda_{abc} = \underline{-\frac{2}{3}f_{bc,a}}$$

$$\square S_{abc} = 0$$



Gravitation polarizations do not grow (stable) !
But the trivial solution is still linearly unstable !

Tensor f_{mn} is a source of instability. Can there exist some regions of instability ?

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Energy-momentum tensor

$$\square R_{\mu\epsilon\nu\tau} = R_{\mu\nu,\epsilon\tau} - R_{\mu\tau,\nu\epsilon} + R_{\epsilon\tau,\mu\nu} - R_{\epsilon\nu,\mu\tau} \quad R_{\mu\nu} \propto \Phi_{(\mu,\nu)}$$

Symmetrical equation does not lead to energy-momentum :

$$\mathbf{E}_{(\mu\nu)} = -G_{\mu\nu} - \frac{2}{3}(\Phi_{(\mu;\nu)} - g_{\mu\nu}\Phi^\lambda{}_{;\lambda}) + (\Lambda^2) = 0$$

Prolonged equation, $\mathbf{E}_{(\mu\nu);;\lambda} = 0$, can be written as RG-gravity:

$$G_{\mu\nu};;\lambda + G_{\epsilon\tau}(2R_{\epsilon\mu\tau\nu} - \frac{1}{2}g_{\mu\nu}R_{\epsilon\tau}) = T_{\mu\nu}(\Lambda'^2, \dots), \quad T_{\mu\nu};\nu = 0$$

$$f_{\mu\nu};\nu = -\frac{1}{2}S_{\mu\nu\lambda}f_{\nu\lambda}$$

$$T_{\mu\nu} = \frac{2}{9}(\frac{1}{4}g_{\mu\nu}f^2 - f_{\mu\lambda}f_{\nu\lambda}) + A_{\mu\epsilon\nu\tau}(\Lambda^2)_{,\epsilon\tau} \leftarrow \boxplus$$

This eq-n can be derived from the (trivial, quasi-) action:

$$L^* = \mathbf{E}_{(ab)}^2 + \varkappa \mathbf{E}_{aa}^2 = C^\mu{}_{;\mu} + \underline{R_{\mu\nu}G^{\mu\nu}} + \frac{1}{9}f_{\mu\nu}f^{\mu\nu} + (\Lambda'\Lambda^2, \Lambda^4)$$

$$G^{\mu\nu}\Phi_{\mu;\nu} \rightarrow (G^{\mu\nu}\Phi_\mu)_{;\nu}$$

Modified gravity as a bonus !!

only f -component (three transverse polarizations in $D=5$) carries D -momentum and angular momentum ('powerful', or tangible waves); other 12 polarizations are 'powerless', or 'intangible' (this is a very unusual feature);

f -component feels only metric and S -field which has effect only on polarization of f : S does not enter eikonal equation, and f moves along usual Riemannian geodesic (if background has $f=0$)

Topological charges and quasi-charges

$$g_{\mu\nu} \Rightarrow \eta_{\mu\nu}, \quad h^a_\mu \Rightarrow \sigma^a_\mu \in SO(1,4) \Rightarrow SO_4$$

$$q^* = a \cdot q \cdot b^{-1}, \quad |a| = |b| = 1$$

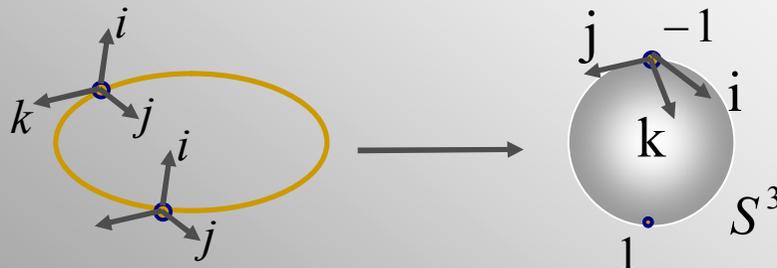
$$q = x_1 \cdot i + x_2 \cdot j + x_3 \cdot k + x_4$$

$$(f, g) \quad \sigma : R^4 \rightarrow SO_4 = S^3 \times S^3 / \pm$$

Group of topological charge:

$$\Pi_0 = \pi_4(SO_4) = Z_2 + Z_2$$

$$f : R^4 \rightarrow S^3$$



arXiv: gr-qc/0610076, gr-qc/0412130

Sym	$\Pi_l(Sym) \rightarrow \Pi_l(Sym^*)$	
1	Z_2	
$SO\{1, 2\}$	$Z_{(e)} \xrightarrow{e} Z_2$	e
$SO\{1, 2\} \times P\{0, 3\}$	$Z_{(v)} + Z_{(H)} \xrightarrow{i, m^2} Z_{(e)}$	$v^0; H^0$
$SO\{1, 2\} \times P\{2, 3\}$	$Z_{(W)} \xrightarrow{0} Z_{(e)}$	W
$SO\{1, 2\} \times P\{0, 2\}$	$Z_{(Z)} \xrightarrow{0} Z_{(e)}$	Z^0
$SO\{1, 2\} \times P\{0, 3\} \times P\{2, 3\}$	$Z_{(\gamma)} \xrightarrow{0} Z_{(H)} \xrightarrow{0} Z_{(W)}$	γ^0

background

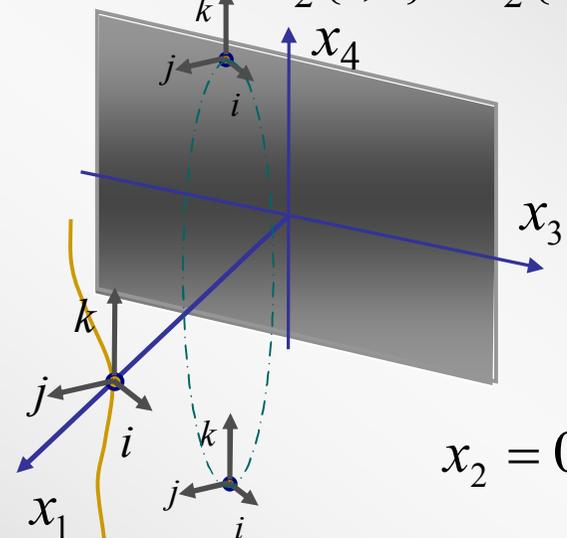
$$\sigma(sx) = s\sigma(x)s^{-1} \quad \forall s \in G \subset SO_3 \times P_4$$

Symmetries and quasi-charges

cylindrical + discrete

$$SO_2\{1, 2\}$$

$$SO_2\{1, 2\} \times P_2\{4, 3\}$$



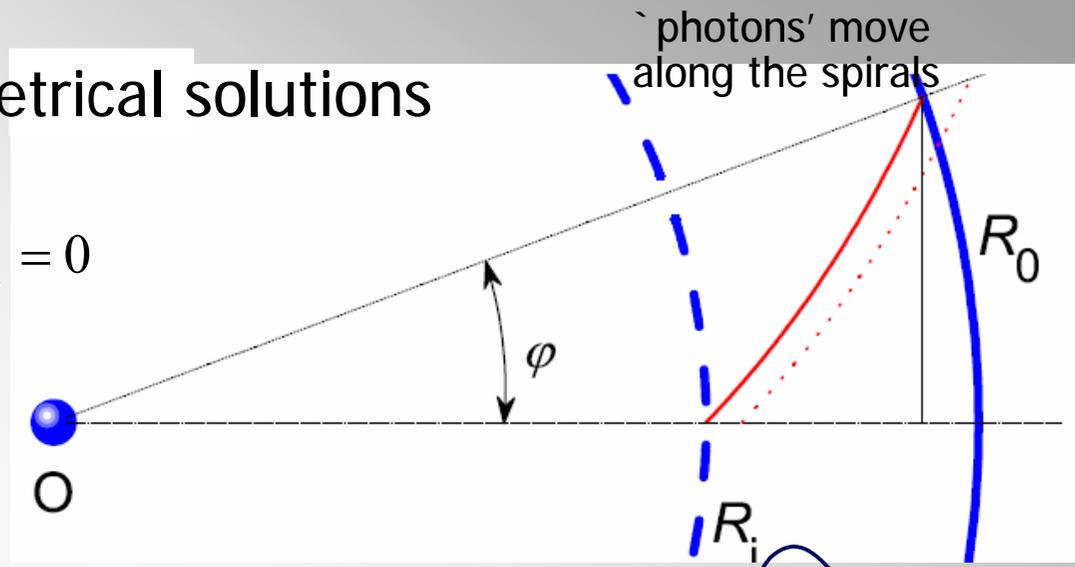
$$x_2 = 0$$

AP Spherically-symmetrical solutions

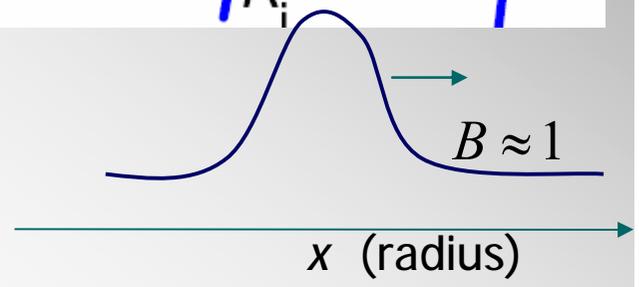
High symmetry: $f_{\mu\nu} = 0, S_{\mu\nu\lambda} = 0$

Scalar responsible for longitudinal wave: $\psi_{,\mu} = \Phi_{\mu}$

Nonuniformity of metric behaves as inhomogeneous refraction
 $f \sim 1/t^{0.5}; a \sim t^{0.5}$



$$\delta f^{\mu\nu}{}_{;\nu} = 0$$



Relativistically expanding S^3 -spherical shell serves as a storage for tangible f -waves (noise) which should move very tangentially to the shell (at very-very small angles). Growing intangible waves can scatter and leave the shell (non-linearity and large fluctuations, even with topological charge+anticharge)

$$\varphi = \frac{1}{\Gamma B} \ln(1+z)$$

$$\Gamma \gg 1 \quad \varphi \ll 1 \quad (\text{for } z_{\text{CMB}} \sim 10^3)$$

Anti-Milne Model: $a = H_0 t, k = +1$

$$\mu(z) = \mu_0 + 5 \log[(1+z) \ln(1+z)]$$

GR

FRW model and Hilbert-Einstein action

Transition to a proxy-lagrangian for the scale factor $a(t)$

$$S_{HE} = - \int dx^4 \sqrt{-g} R \Rightarrow S_1 = \int dt a \dot{a}^2$$

$$a = qt^{2/3}$$

AP

Lagrangian 4D-phenomenology for topological quanta

Can we write a 4D proxy-lagrangian (holography)?

$$S_{5D} = \int dx^5 \sqrt{-g} (R_{\mu\nu} G^{\mu\nu} + \frac{1}{9} f_{\mu\nu} f^{\mu\nu})$$

Additional (classical) fields and constraints?

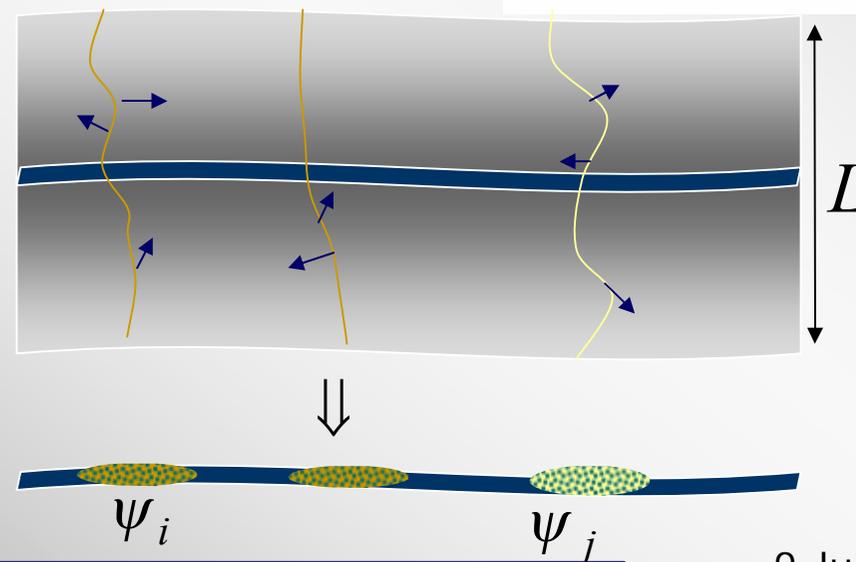
$$\Downarrow \times \langle L \rangle$$

$$S_{4D}^* = \int dx^4 \sqrt{-g} (R_{\mu\nu} G^{\mu\nu} + \sigma R + L_{\text{quanta}}(\psi_i, \psi'_i; L, h_0, h_{i.n.}, h_{t.n.}, \dots))$$

$$\sigma = \langle R \rangle_{i.n.} \propto \langle \Lambda^2 \rangle_{i.n.}$$

Can it look like a 4D QFT on a classical background?

Projection along the extra dimension on the central layer (surface); high anisotropy of tangent waves (tangible noise) enable superposition of proxy-fields (psi-filds)

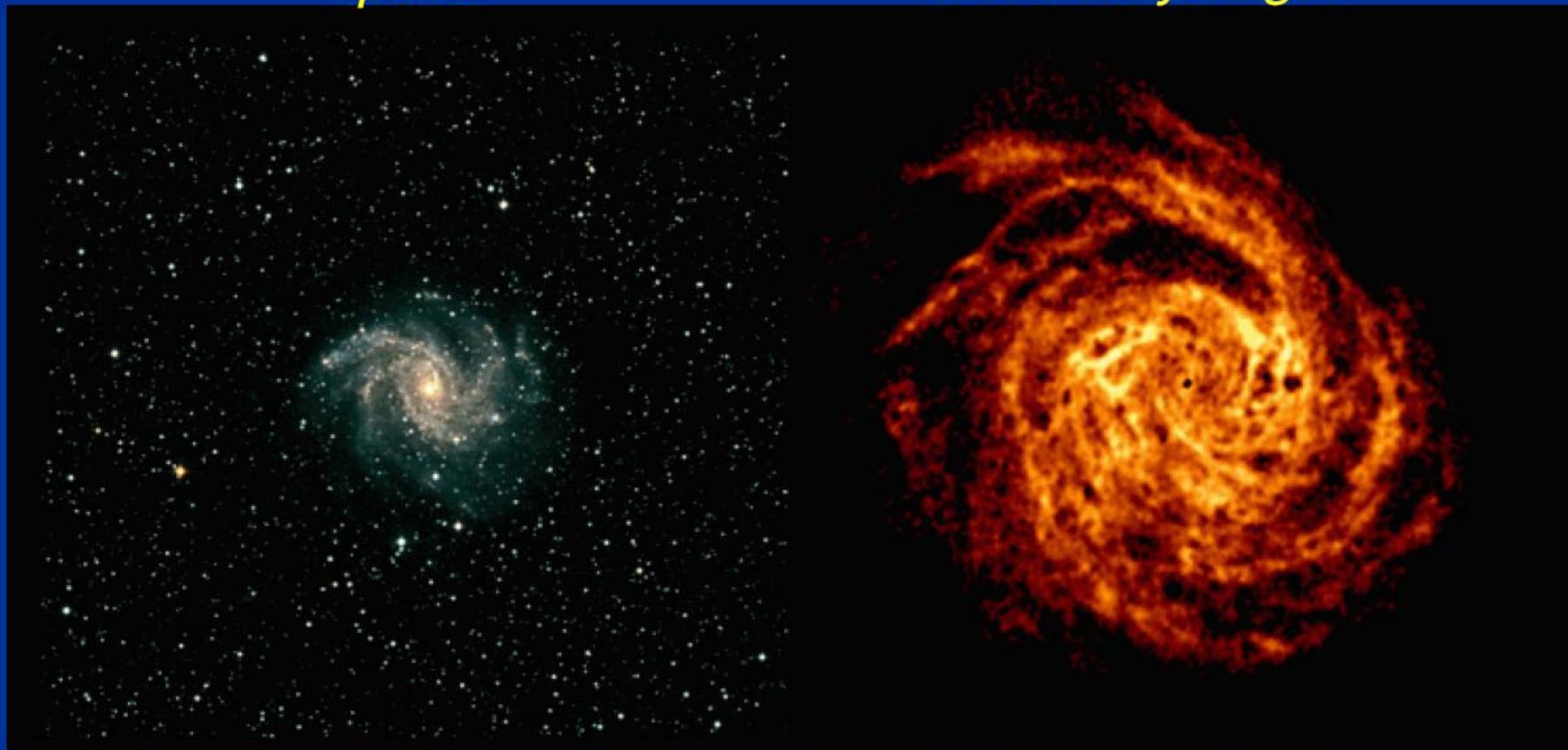


NGC 6946

*Strong HI outer arms
Strong velocity wiggles
HI holes; High velocity clouds*

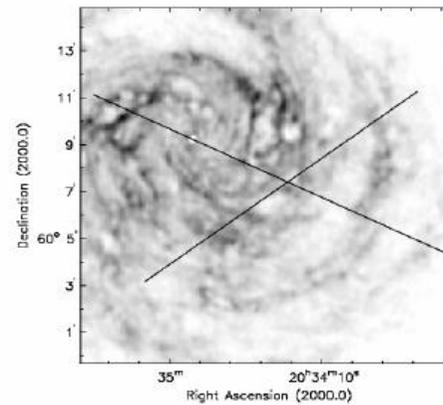
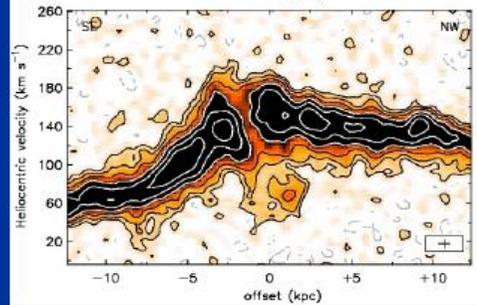
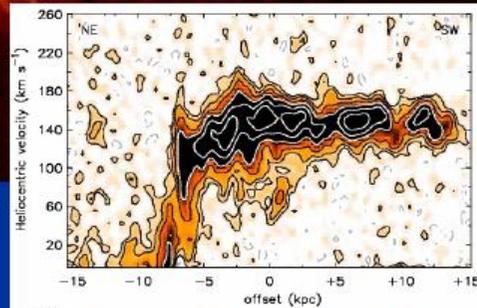
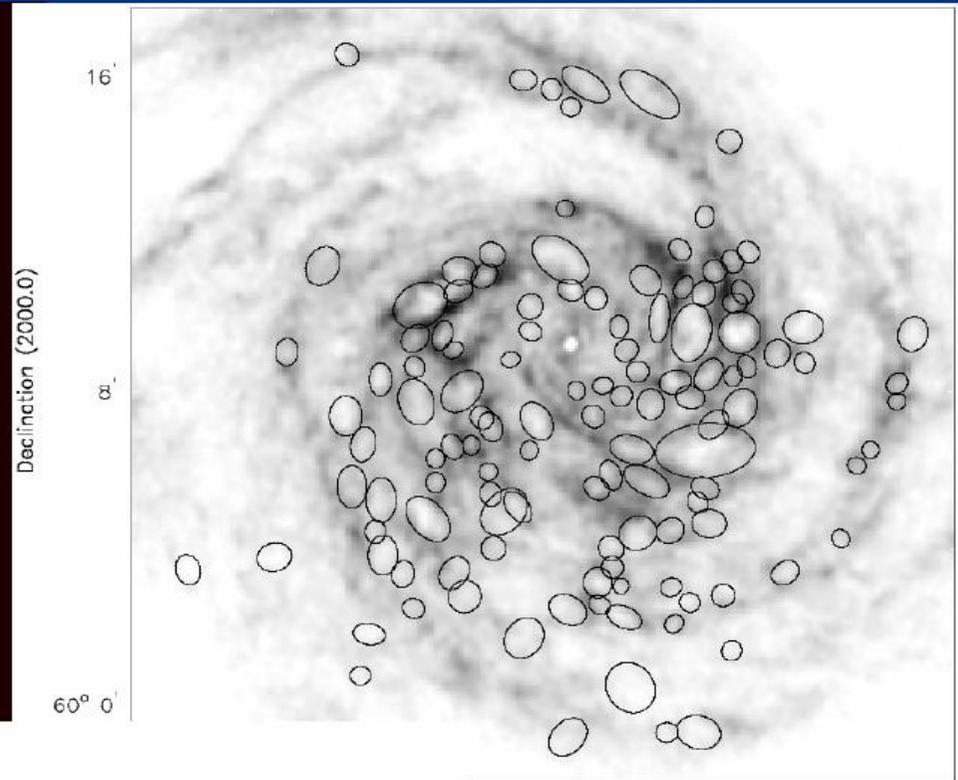
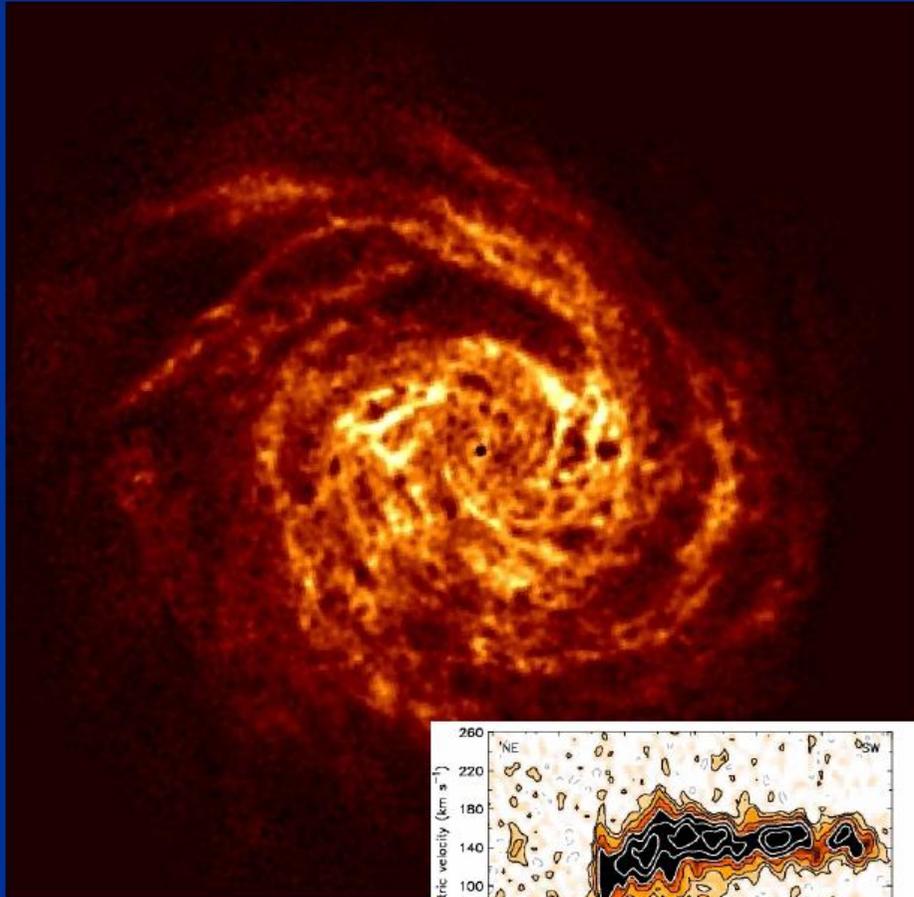
optical

neutral hydrogen



same scale

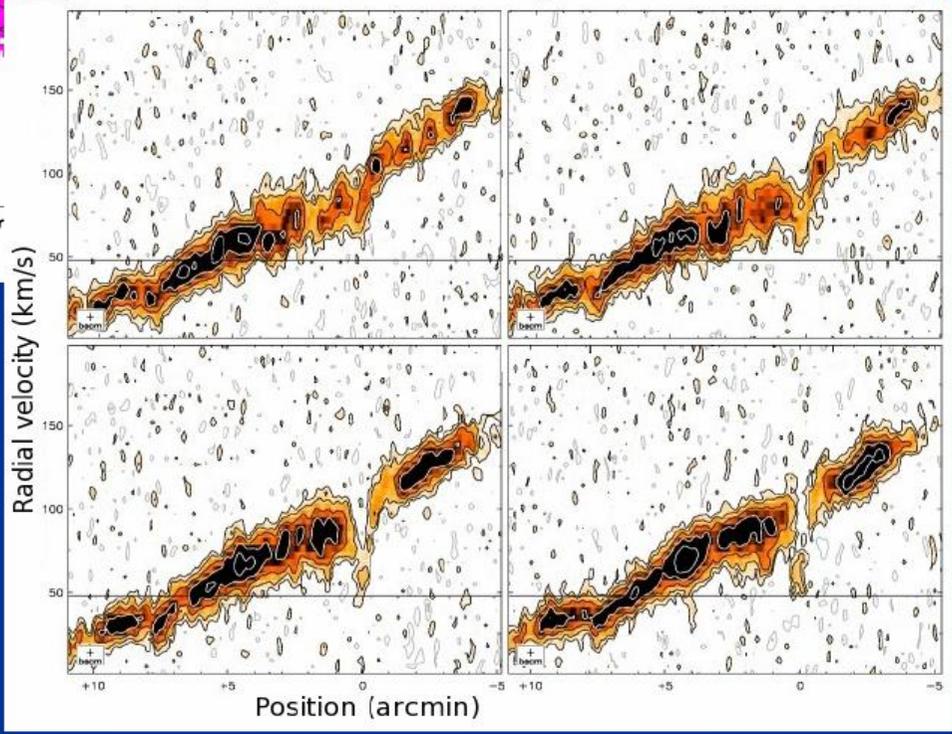
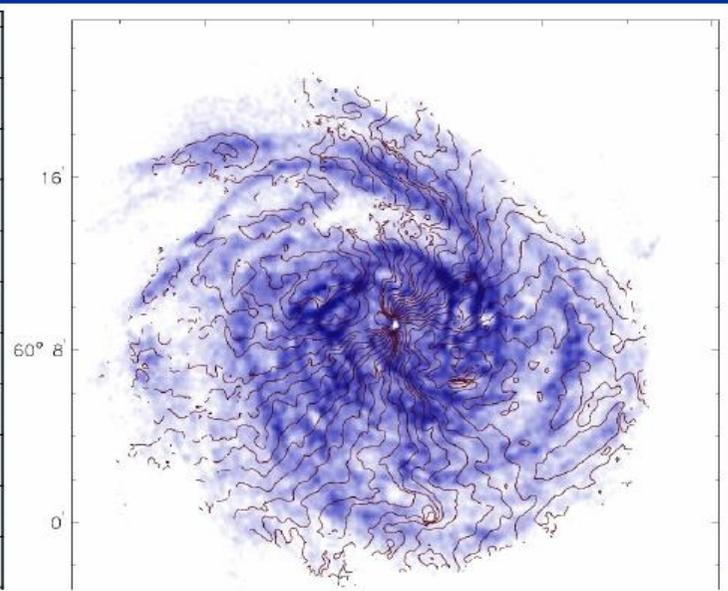
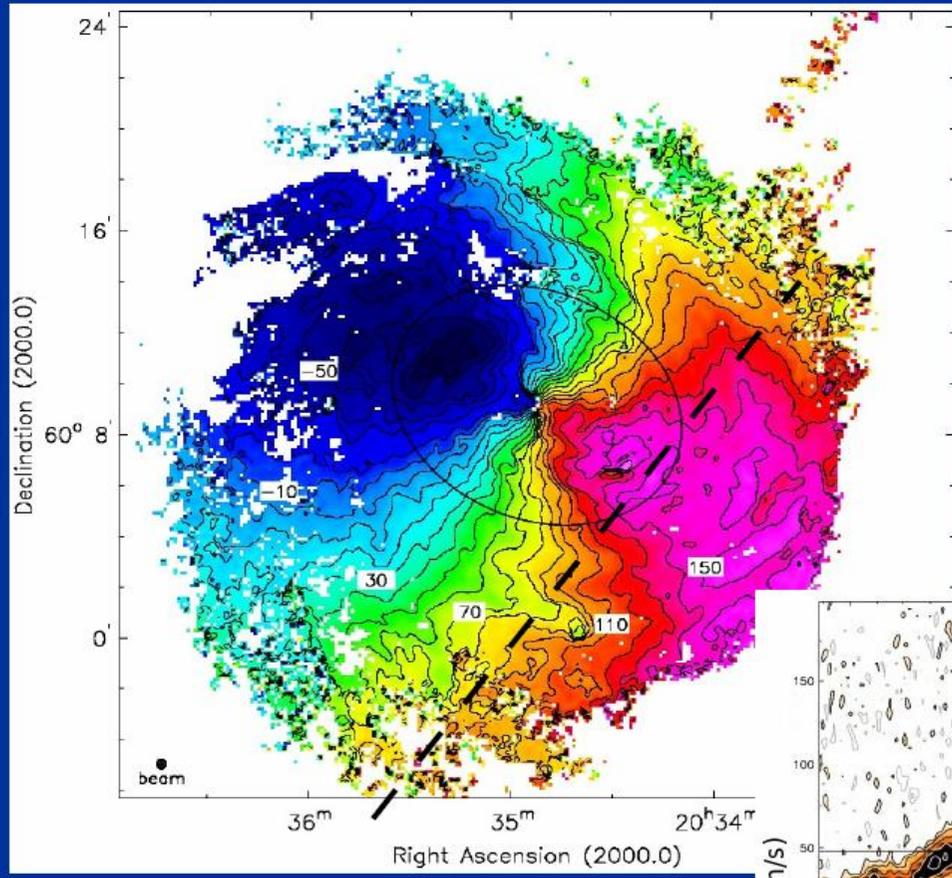
Boomsma et al. 2007



35^m 20^h34^m10^s
 Ascension (2000.0)

**high
 velocity
 clouds**

HI holes



Velocity wiggles

Light traces mass

WHY?

Easy way to change

$$\frac{1}{r^2} \rightarrow \frac{1}{r}$$

Plain RG-gravity in 5D

extra gravity instead of extra mass

$$\sqrt{-g} R_{\epsilon\tau} G^{\epsilon\tau}$$



$$G^{\mu\nu}{}_{;\lambda;\tau} g^{\lambda\tau} + G_{\epsilon\tau} (2R^{\epsilon\mu\tau\nu} - \frac{1}{2}g^{\mu\nu} R^{\epsilon\tau}) = T^{\mu\nu}$$



$$\Delta^2 \varphi = -\frac{a}{R^3} \delta(R)$$



$$\varphi(R^2) = \frac{a}{8} \ln R^2 - \frac{b}{R^2} (+c); \quad F_{\text{point}} = \nabla \varphi = \frac{a}{4R} + \frac{2b}{R^3}$$

The admixture of Einstein-Hilbert term
adds more complexity (Yukawa motives)

masses are distributed along
the extra dimension

$$\mu_1(p) = \pi^{-1} / (1 + p^2)$$

$$\int \mu(p) dp = 1$$

$$\frac{1}{r^2} \longrightarrow \frac{1}{r}$$



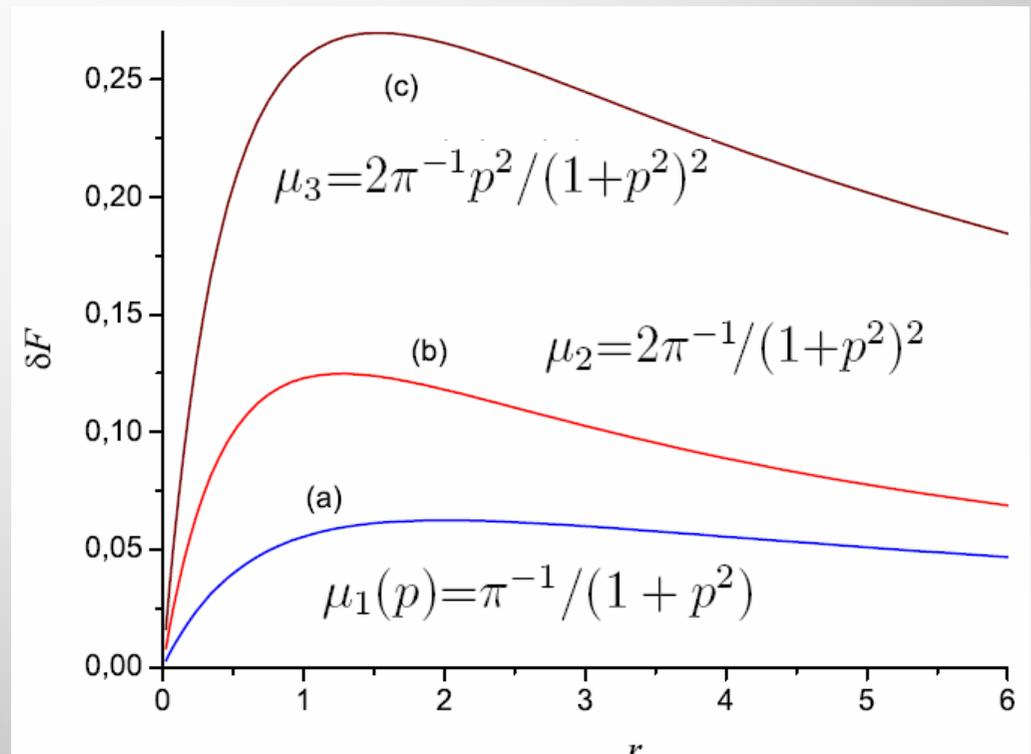
$$F(r) = \frac{a}{8 + 4r} + \frac{2b(1 + r)}{r^2(2 + r)^2}$$

↓ $a = b = 2$
to make initial
correction zero

$$F(r) = \frac{1}{r^2} + \frac{r}{2(2 + r)^2}$$

5D RG-gravity

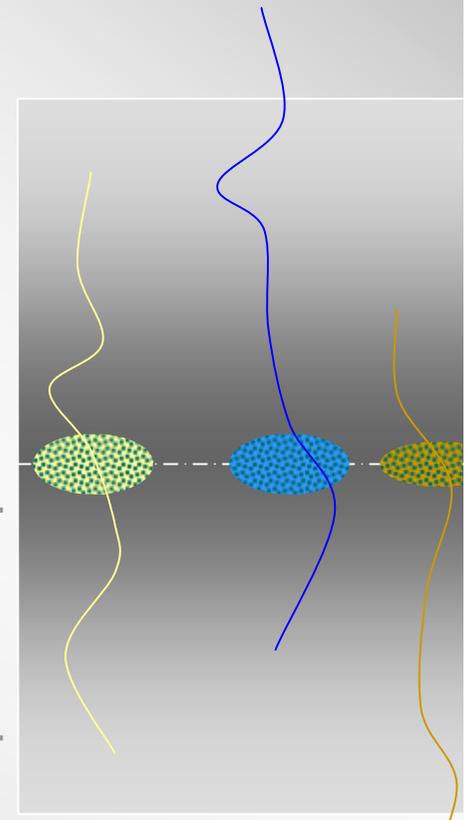
dimensionless deviation from the
Newton's Law $(F/F_N - 1)L^2/r^2$ as a
function of r/L



Several conclusions

- Riemann-squared modified gravities are not appropriate
- AP grants beautiful and simple eq-n: no singularities; no free parameters ($D = 5$ is a must); topological quanta with a $4D$ -phenomenology looking like QFT (on a classical background, with a modified gravity)
- Can this mathematical reality coincide with our Universe? – Maybe. Some qualitative predictions are possible: seemingly, no spin zero elementary quanta, no more than four generations (lepton flavors); neutrinos are true neutral (kind of Majorana); no room for supersymmetry and DM
- So, we are still waiting for LHC
(`theory' without falsifiable predictions is too safe !!)

Thank you for your attention !



Anti-Milne model
arXiv: 0902.4513

$$\mu(z) = \mu_0 + 5 \log[(1+z) \ln(1+z)]$$

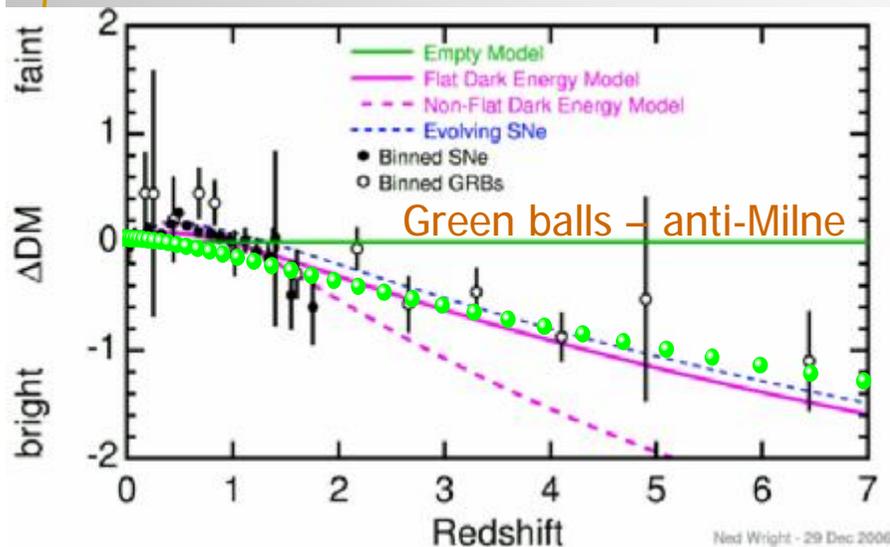
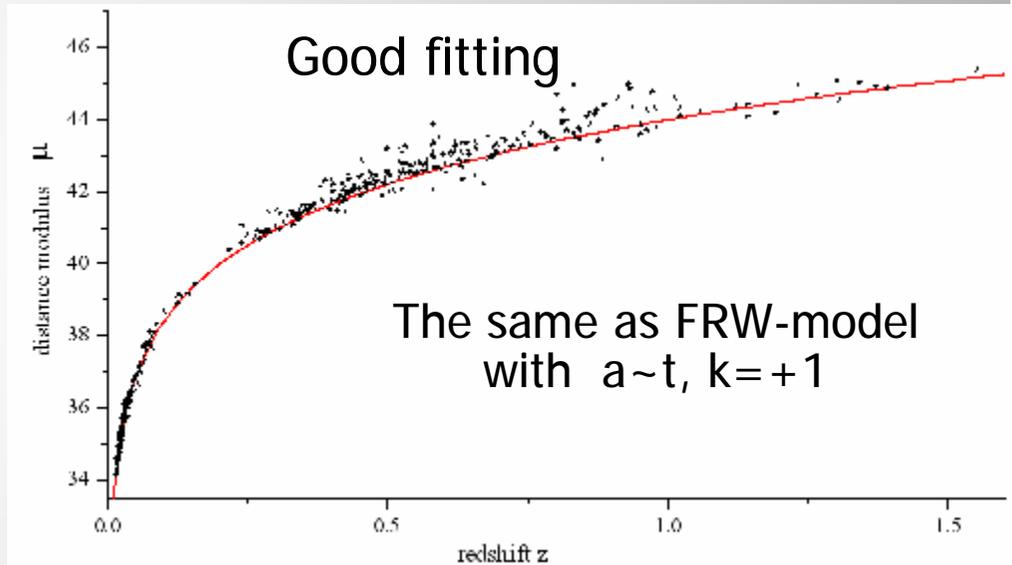
$$\mu_0 = -5 \log(H_0 d_*/c) \approx 43.3 \quad d_* = 10 \text{ pc}$$

$$\mu(z) = \mu_0 + 5 \log(z + z^2/2)$$

Milne model (or empty model)
- as the reference point here

SNe Ia data from Hicken, et al.
arXiv: 0901.4804

$$H_0 = t_0^{-1} = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$$



Thank you again!