of Solutions

SINGULARITIES IN GRAVITATIONAL THEORY

I. L. Zhogin

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E-mail: zhogin@mail.ru

1. Two methods are known for analysis of singularities in the equations of the general theory of relativity. Topological methods (Penrose, Hawking [1, 2]) demonstrate the unavoidability of singularities, but say little about their structure or how they can be traced, if they can be. On the other hand the analytical character of general relativity solutions near a singularity has been studied by Belinskii, Lifshits, and Khalatnikov [3]. But it remains unclear whether the impossibility of extending the solution beyond the singularity was related to the coordinate system (synchronous) chosen in [3].

We will arbitrarily term a singularity as "weak," if the solution can be extended beyond it, and "strong" in the opposite case; the appearance of a "strong" singularity apparently indicates some internal limitation of the corresponding equations.

Below we will consider the theory of absolute parallelism and demonstrate that among the ~~composite~~ compatible equations ($h_\mu{}^a$ n-~~references~~ frame) of the Einstein–Mayer classification [4] (including the vacuum general relativity equation), only one is free of "strong" singularities. The notation used is described in [5]; in particular,

$$g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu = h_{a\mu} h_{a\nu};\ \Lambda_{a\mu\nu} = 2h_{a[\mu;\nu]};\ \Lambda_{a[\mu\nu;\lambda]} \equiv 0;$$
$$S_{\mu\nu\lambda} = 3\Lambda_{[\mu\nu\lambda]};\ \Phi_\mu = h_a{}^\lambda \Lambda_{a\lambda\mu} = \Lambda_{\lambda\lambda\mu};\ f_{\mu\nu} = 2\Phi_{[\mu;\nu]};\ {}_{,a} = {}_{;\mu} h_{a\mu}. \tag{1}$$

We will omit η^{ab} (and $g^{\mu\nu}$ if derivatives are covariant everywhere) in the ~~convolutions~~ contractions.

The formal (local and covariant) method for defining the type of singularity is related to verifying the ~~consistency~~ compatibility of the equations (i.e., the possibility of constructing the formal solution in the form of a series, [6]) under limiting conditions where the matrix $h_\mu{}^a$ is degenerate.

Thus, after a substitution of the form $h_\mu{}^a = HPH_\mu{}^a$ ($H = \det H_\mu{}^a$) the main portion of any system of equations [4] can be represented in a form 3-linear relative to $H_a{}^\mu$ [5] (i.e., the coefficients of the main derivatives $H_{a,\nu\lambda}^{\mu}$ are quadratic in $H_a{}^\mu$). Regularity of the main portion is maintained if $H_a{}^\mu$ is degenerate and finite and $r = \operatorname{rank} H_a{}^\mu \geq 2$, but in the general case within the remaining terms one cannot eliminate the inverse matrix $H_{a\mu}$, i.e., they are singular, ~~vanishing at~~ going to infinity if $r < n$. Such a situation, where the main portion is regular (number of equations is the same and a general solution can be constructed), while the non-main portion is singular, corresponds as far as can be seen to breakoff of the solution at the singularity; this proposition can be confirmed by examples from the field of ordinary differential equations.

However, there does exist a class of single-parameter equations [5]

$$E_{ab} = \Lambda_{abc,c} - \sigma\Phi_{a,b} + 2\sigma\Phi_{b,a} - \sigma\eta_{ab}\Phi_{c,c} + U_{ab}(\Lambda^2) = 0, \tag{2}$$

$U_{ab} = \Lambda_{acd}\Lambda_{cdb} + 3\sigma\Lambda_{abc}\Phi_c + 2\sigma^2(\Phi_a\Phi_b - \eta_{ab}\Phi^2)$, which after the transformation referred to have a trilinear form: the non-main terms do not contain $H_a{}^\mu$. Within the solutions of Eq.

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(2) there appear "weak" singularities with $r < n$ [5], while for a number of σ values and dimensionalities in the field $h_\mu{}^a$ remains real beyond the singularity.

In the present note we will consider another possibility for Eq. (2), another channel for appearance of singularities, where the matrix $h_{a\mu}$ remains finite and degenerates (cosingularities).

2. We will introduce k-minor notation (minors of rank ~~range~~ n-k) for $h_\mu{}^a$ and $g_{\mu\nu}$:

$$\begin{pmatrix}\mu_1 \dots \mu_\kappa \\ a_1 \quad a\end{pmatrix} = \partial^\kappa h/\partial h^{a_1}{}_{\mu_1}\dots\partial h^{a_\kappa}{}_{\nu_\kappa} = \kappa!\, h h^{\mu_1}_{[a_1}\dots h^{\mu_\kappa}_{a_\kappa]};$$

$$[\mu_1\nu_1,\ \dots,\ \mu_\kappa\nu_\kappa] = \partial^\kappa(-g)/\partial g_{\mu_1\nu_1}\dots\partial g_{\mu_\kappa\nu_\kappa} = \frac{1}{\kappa!}\begin{pmatrix}\mu_1\dots\mu_\kappa \\ a_1 \quad a_\kappa\end{pmatrix}\begin{pmatrix}\nu_1\dots\nu_\kappa \\ a_1 \quad a_\kappa\end{pmatrix};$$

which minors, as well as the determinant, are polylinear combinations of ~~equations~~ for the elements of $h_\mu{}^a$ (or $g_{\mu\nu}$). We may thus change the form of Eq. (2) such that the coefficients for arbitrary $h_{a\mu,\nu\lambda}$ will be quadratic in 2-minors of the matrix $h_\mu{}^a$. For example, for the variant $\sigma = 0$ we easily obtain

$$h^2 E_a{}^\mu = h^2 \Lambda_a{}^{\mu\nu}{}_{,\nu} + \dots = h_{a\alpha,\beta\nu}[\alpha\mu,\ \beta\nu] + (h'^2). \tag{3}$$

By coordinate transforms [and rotation of Eq. (9)] the fin~~al~~ite degenerate matrix can be reduced to the form $h_\mu{}^a = \mathrm{diag}(1, \dots, 1, 0, \dots, 0)$, after which it can be proven that for $r^* = \mathrm{rank}\, h_\mu{}^a = n - 1$ regularity of the main portion of Eq. (3) is preserved [if $r^* \le n - 2$, the main terms vanish in a portion of Eq. (3)]. [or rank $g_{\mu\nu} \le n-2$]

Assuming the necessity of the replacement $h_\mu{}^a = H^p H_\mu{}^a$, we write Eq. (2) in the form

$$\tilde{E}_a{}^\mu = (a_1 E_{ab} + a_2 E_{ba} + a_3 \eta_{ab} E_{cc}) H_b{}^\mu H^{2p} = 0. \tag{4}$$

Having taken $H_\mu{}^a = \mathrm{diag}(1, \dots, 1, \delta)$, from the requirement of maintaining terms of order $\sim 1/\delta^4$, $1/\delta^3$ in the main portion (these appear in all the equations therefore must be maintained) we define constants (the general coefficient being a matter of taste) $a_1 = 1 - 2\sigma$, $a_2 = -\sigma$, $a_3 = 0$, $p = 0$ (for $\sigma = 1/3$ the value of p is undefined).

We can now write the equations (main portion) in the desired form

$$h^2\tilde{E}_a{}^\mu = h_{b\alpha,\beta\nu}\left((1-2\sigma)\eta_{ab}[\alpha\mu,\ \beta\nu] - \sigma\begin{pmatrix}\alpha\beta\\ac\end{pmatrix}\begin{pmatrix}\mu\nu\\bc\end{pmatrix} - \sigma(1-3\sigma)\begin{pmatrix}\alpha\beta\\bc\end{pmatrix}\begin{pmatrix}\mu\nu\\ac\end{pmatrix}\right) + \dots \tag{5}$$

when(?)

The main portion is regular if $r^* = n - 1$, aside from the cases $\sigma = 1$ and $\sigma = 1/3$, while Eqs. (4) and (5) contain respectively only the symmetric and antisymmetric portions of system (2), which require individual examination.

First, we must turn to the equation $h^2\tilde{E}_a{}^\mu h_a{}^\nu = 0$, with a 3-minor appearing in the main terms. For the variant $\sigma = 1$ it follows from Eq. (5) that

$$h^2\tilde{E}_a{}^\mu h_a{}^\nu = -\frac{1}{2}\left(\begin{pmatrix}\mu\lambda\\cd\end{pmatrix}\begin{pmatrix}\nu\alpha\beta\\bcd\end{pmatrix} + \begin{pmatrix}\nu\lambda\\cd\end{pmatrix}\begin{pmatrix}\mu\alpha\beta\\bcd\end{pmatrix}\right) h_{b\alpha,\beta\lambda} + (h'^2). \tag{6}$$

For $\sigma = 1$ the symmetric equation $E_{(\mu\nu)}$, see Eq. (2), coincides [5] in its main portion with the general relativity equation, allowing the following representation:

$$-4gP^{\mu\nu} = -[\mu\nu,\ \varepsilon\tau,\ \alpha\beta]R_{\varepsilon\alpha\tau\beta} = 2[\mu\nu,\ \varepsilon\tau,\ \alpha\beta](g_{\alpha\beta,\varepsilon\tau} + g^{\rho\varphi}\Gamma_{\rho,\varepsilon\tau}\Gamma_{\varphi,\alpha\beta}) = 0. \tag{7}$$

The main portion of Eq. (6) is regular, if $r^* \ge n - 1$ [and in Eq. (7), if $\rho = \mathrm{rank}\, g_{\mu\nu} \ge n - 1$, but the remaining terms diverge at $\rho < n$, the case of "strong" singularities]. We can try to express the antisymmetric portion in a form other than Eq. (4), in other indices. In doing this we cannot avoid 1-minors, but nevertheless for the equation ($H_{ab} = 2E_{[ab]}$, see [5], $\sigma = 1$)

$$h^2 H_{ab} = h_{c\mu,\nu\lambda}\begin{pmatrix}\lambda\\d\end{pmatrix}\left(\eta_{ac}\begin{pmatrix}\mu\nu\\bd\end{pmatrix} - \eta_{bc}\begin{pmatrix}\mu\nu\\ad\end{pmatrix} + 3\eta_{cd}\begin{pmatrix}\mu\nu\\ab\end{pmatrix}\right) + (h'^2) = 0\,. \tag{8}$$

The main portion is regular for $r^* \ge n - 1$, as for system (6), (8) as a whole. In calculating the equations (main portion) the signature δ_{ab} and indices a, $\mu = 1, \dots, n$ will be more convenient. Taking $h^a{}_\mu=\delta_{a\mu}-\delta_{na}\delta_{n\mu}$, $\binom{\mu}{a}=\delta_{na}\delta_{n\mu}$ etc., it is simple, for example, to show that the main portion of the component in (i < n) from Eq. (8) vanishes and does not coincide with the analogous equation from Eq. (6).

The fact that the non-main terms in Eqs. (5) and (8) diverge ($\delta \neq 1/3$) when $r^* < n$ can be shown by writing the equation (in which the last term ~1/h; again, "strong" singularities)

$$h^3H^{\mu\nu}\left(\sim h^2\tilde{E}_a^{[\mu}\binom{\nu]}{a},\ \sim h^2H_{ab}\binom{\mu\nu}{ab}\right)=h\,(h^2S^{\mu\nu\lambda})_{,\lambda}-h^2S^{\mu\nu\lambda}\,(h_{,\lambda}+$$

$$+(1-3\sigma)\,h\Phi_\lambda)-(1-3\sigma)\,[\mu\alpha,\ \nu\beta]\left(\binom{\lambda}{b}h_{b\alpha,\beta,}-\frac{1}{h}\binom{\varepsilon}{b}\binom{\tau}{c}h_{b\alpha,\tau}\,h_{c\varepsilon,\beta}\right)=0,$$

see (13), $h^2S^{\mu\nu\lambda}=1/2\binom{\alpha\beta}{bc}\binom{\mu\nu\lambda}{abc}h_{a\alpha,\beta,}\ \ h\Phi_\lambda=h_{,\lambda}-\binom{\varepsilon}{b}h_{b\lambda,\varepsilon}.$

The most interesting variant of Eq. (2) remains: $\sigma = 1/3$. For the antisymmetric portion it is simple to obtain

$$H^{\varkappa}H^{\mu\nu}=1/2\binom{\alpha\beta}{bc}\binom{\mu\nu\lambda}{abc}H_{a\alpha,\beta\lambda}+(H'^2),\ \ \varkappa=4p+2.$$

Here the minors refer to the matrix $H_\mu{}^a$. The value of p can be chosen such that the remaining terms will not be singular. But the symmetric equation cannot be written in this form while maintaining regularity in the case $r^* = n - 1$ (the number of components in the symmetric portion is too large, and the 1-minors vanish too easily). Therefore, this second channel for singularity development is impossible in the case $\sigma = 1/3$.

Thus, within absolute parallelism one can choose (a priori) a unique system of equations using the requirement of absence of "strong" singularities, which apparently lead to incompleteness of space–time.

3. We will add to the above a few words in favor of choosing the absolute parallelism geometry (this theory involves only the choice of the field $h_\mu{}^a$). For trivial solutions ($\Lambda = 0$) the meaning of the field is evident. In this case a potentials y_a can be introduced (inertial coordinates), integrating the equations $y_{a,\mu} = h_{a\mu}$ ($\Lambda_{a\mu\nu} = 0$), and the vector fields $\xi_a = h_a{}^\mu\partial_\mu$, $\zeta_{ab} = 2y_{[a}\xi_{b]}$, for which the Lie equations corresponding to a Poincare group are satisfied:

$$[\xi_a,\ \xi_b]=0,\ [\xi_a,\ \zeta_{bc}]=2\eta_{a[b}\,\xi_{c]},\ [\zeta_{ab},\ \zeta_{cd}]=2\eta_{d[a}\,\zeta_{b]c}-(cd).$$

These equations are invariant relative to the global transforms (which form symmetry group of inertial coordinates)

$$h^{*a}{}_\mu=\kappa s^a{}_b\,h^b{}_\mu(x),\ s^a{}_b\in O(1,\ n-1),\ \kappa>0;\ \kappa,\ s^a{}_b=\text{const.} \tag{9}$$

If we require that the field equations $h_{a\mu}$ also allow the replacement of Eq. (9), this leads at once to the absolute parallelism theory. Then the near vicinity of any arbitrary point (if we neglect derivatives h') appears like a Minkowsky space. In other words, without departing from the vicinity or differentiating one cannot construct an invariant or scalar which would distinguish one point of space from another; the points themselves are indistinguishable. This is a natural property, reducing the possibility of fantasy (and, surely, of singularities).

Invariance relative to the rotations of Eq. (9) can be insured if we use η_{ab} in operations with scalar indices. Invariance relative to "scale" transforms (involving coordinates y_a) of the transforms of Eq. (9) also affects the form of the equations and allows determination of the "mathematical dimensionality" of the quantities, according to the power of k with which they transform, for example, $h_{a\mu} \sim k$, $h_a{}^\mu \sim 1/k$ (η_{ab} is "dimensionless"), $\Lambda_{abc} \sim 1/k$, $\Lambda_{abc,d} \sim 1/k^2$. Therefore, to equations with the form of Eq. (2) one cannot add terms $(\Lambda_{abc})^4$. One can transform to conventional physical dimensions (lengths) if the transform is accompanied by the replacement $x^\mu \to kx^\mu$.

In the general theory of relativity (excluding the R^2-version and the λ-term) one can also relate dimensionality (which in its essence is nonlocal) to global conformal transforms, but is natural not to disrupt the group of Eq. (9). The global nature of the rotations of

Eq. (9) in absolute parallelism permits determination of nonlocal discrete information — topological (quasi-) charges [7].

E-m tensor

4. We will briefly consider the possibility of determining the energy-momentum tensor. If we compose from the metric (curvature) the invariant M (of definite dimensionality), then by varying ~~its~~ by metric, we obtain a symmetric tensor $D_{\mu\nu}$ and the identity (see [3]):

with respect to

$$\delta(hM)/\delta g_{\mu\nu} = hD^{\mu\nu}\ (h = \sqrt{-g}),\ D_{\mu\nu;\nu} \equiv 0. \tag{10}$$

Now, by applying Eq. (2), we must eliminate from $D_{\mu\nu}$ the linear terms: $D_{\mu\nu} = T_{\mu\nu}(\varepsilon^2)$ ("in the equations" we ~~take~~ obtain $T_{\mu\nu;\nu} = 0$) and verify for weak fields $h_{a\mu} = \eta_{a\mu} + \varepsilon_{a\mu}$ the definition of the sign of the energy $\int T_{00}(\varepsilon^2)dV$. Thus, $M^{(0)} = R$ ($D_{\mu\nu}{}^{0} = -P_{\mu\nu} = g_{\mu\nu}R/2 - R_{\mu\nu}$) does not succeed (at $\sigma = 1$ the linear terms are eliminated [5], but sign definition is lacking) and we must turn to $M \sim R^2$. Commencing from the Bianci identities, we obtain

$$\begin{gathered} D^{(1)}_{\mu\nu} = P_{\mu\nu;\lambda;\lambda} + P_{\varepsilon\tau}(2R_{\varepsilon\mu\tau\nu} - 1/2 g_{\mu\nu} R_{\varepsilon\tau})\ (= T_{\mu\nu}(\Lambda'^2,\ \ldots)); \\ D^{(2)}_{\mu\nu} = R_{;\mu;\nu} - g_{\mu\nu} R_{;\lambda;\lambda} - RR_{\mu\nu} + 1/4 g_{\mu\nu} R^2; \\ D^{\mu\nu}_{(3)} = (-g)^{-1} |\mu\nu,\ \alpha\beta,\ \gamma\delta,\ \varepsilon\tau,\ \rho\varphi|\, R_{\alpha\gamma\beta\delta} R_{\varepsilon\rho\tau\varphi}\ (n \geqslant 5). \end{gathered} \tag{11}$$

Since $D_{\mu\mu}{}^{(1)} = (2 - n/2)P_{\varepsilon\tau}R_{\varepsilon\tau} + C_{\mu;u}$, it is clear that $M^{(1)} = -P_{\varepsilon\tau}R_{\varepsilon\tau}$. With consideration of the equation $E_{\mu\mu}$ we obtain (approximate equations exact for terms of order $\sim\varepsilon^2$)

$$R = r(\Lambda^2),\ D^{(2)}_{\mu\nu} \approx (r(\eta_{\mu\tau}\eta_{\varepsilon\nu} - \eta_{\mu\nu}\eta_{\varepsilon\tau}))_{,\varepsilon\tau} = A^{(2)}_{\mu\varepsilon\nu\tau}(\varepsilon^2)_{,\varepsilon\tau}.$$

Since $D_{\mu\nu,\nu}{}^{(3)} \cong 0$ it is understood that $D^{(3)}$ also reduces to the trivial form $D_{\mu\nu}{}^{(3)} = A_{\mu\varepsilon\nu\tau}{}^{(3)}(\varepsilon^2)_{\varepsilon\tau}$ ($A_{\mu\varepsilon\nu\tau}$ has Riemann tensor symmetry); such terms produce no contribution to the n-impulse and its moments.

If we eliminate linear terms from $D_{\mu\nu}{}^{(1)}$ and then using linearized equations separate trivial terms in $T_{\mu\nu}$ so that there remain terms of the type Φ'^2, then in the final outcome $T_{\mu\nu}$ from Eq. (11) reduces to the form

$$T_{\mu\nu} \approx (\eta_{\mu\nu} f_{\varepsilon\tau} f_{\varepsilon\tau} - 4 f_{\mu\varepsilon} f_{\nu\varepsilon})(\sigma - 1)^2/8 + A^{(1)}_{\mu\varepsilon\nu\tau}(\varepsilon^2)_{,\varepsilon\tau}. \tag{12}$$

This result can more easily be proved by transforming the derivative $\delta(hL)/\delta h_\mu{}^a = 2hT_a{}^\mu$; $L = M^{(1)}$. We can remove from L terms

1) of the type $C_{\mu;\mu}$ [to the density hL these produce the trivial contribution $(hC^\mu)_{,\mu}$];

2) of the form $\Lambda^2\Lambda'$, Λ^4 (invariant!), since their contribution ($\sim\varepsilon^2$) is trivial: $\Delta(hT_a{}^\mu) = B_\alpha{}^{\mu\nu}{}_{,\nu}$, $B_{a(\mu\nu)} = 0$; for $\Delta T_{a\mu}$ there exists an identity of the form of Eq. (10);

3) quadratic in the equations (of the type E^2), since the variation is linear relative to differentiation with respect to field.

In this sense an equivalence $L \simeq (\sigma - 1)^2/4 f_{\varepsilon\tau}f^{\varepsilon\tau} = \tilde{L}$ is possible. Variation of $h\tilde{L}$ with respect to $\Phi_{\varepsilon,\tau}$ yields terms which after application of Maxwell equation (14) reduce to trivial form $B_{a\nu\mu}(\varepsilon^2)_{,\nu}$ ($B_{a(\mu\nu)} = 0$), while variation over the metric yields the electromagnetic contribution of Eq. (12).

If we go further and consider the invariants $M \sim R'^2$, R''^2, etc., we can demonstrate their equivalence to zero using the method above.

Thus, there exists an energy-momentum tensor $T_{\mu\nu}$ ($T_{\mu\nu;\nu} = 0$), as well as an approximate conservation law $T^{\mu\nu}{}_{,\nu} = 0$, while the role of the components $f_{\mu\nu}$ stand out: weak fields with $f = 0$ do not transport n-momentum or moments.

We will note that for the equations of R^2-gravitation $C_A D_{\mu\nu}{}^A = 0$ ($0 \leq A \leq 3$; $C_A = \mathrm{const}$; they are irregular for a number of C_A values) one can also evaluate the properties of singularities if the main part is divided in the following manner:

$$g^2 C_A D_A^{\mu\nu} = (C_1/2\, [\mu\nu,\ \varepsilon\tau,\ \alpha\beta]\, [\gamma\delta] + C_2\, [\mu\nu,\ \alpha\beta]\, [\varepsilon\tau,\ \gamma\delta]\, g_{\varepsilon\tau,\alpha\beta\gamma\delta} + \ldots .$$

It can be shown that the remaining terms diverge if rank $g_{\mu\nu} < n$.

5. In the theory considered one cannot eliminate (by coordinate transformation) all terms h' at a point — a portion of these form a tensor (but possibly $g' = 0$); moreover, the

connection
theory can be formulated using asymmetric ~~compendency~~, its very name being related to this fact. Therefore, the question of how a small disturbance (high frequency packet) moves, given some background solution is valid. The antisymmetric portion of Eq. (2) and Maxwell's equation (a differential result of Eq. (2), see [5]), appear as follows:

$$H_{\mu\nu} = 2E_{[\mu\nu]} = S_{\mu\nu\lambda;\lambda} + (1-3\sigma)(f_{\mu\nu} - S_{\mu\nu\lambda}\Phi_\lambda) = 0; \tag{13}$$

$$f_{\mu\nu;\nu} = J_\mu;\ J_\mu = (S_{\mu\nu\lambda}\Phi_\lambda)_{;\nu} = -1/2 S_{\mu\nu\lambda} f_{\nu\lambda} + (1-3\sigma) f_{\mu\nu}\Phi_\nu. \tag{14}$$

We will choose the background solution $h_{a\mu}(x)$ such that $f_{\mu\nu} = 0$ (which is possible for Eq. (2) [5]), and $S_{\mu\nu\lambda} = 0$ as well, which is possible and even necessary in the presence of high symmetry, for example, spherical [5]. In the general case one cannot add to Eq. (2) the equation $S = 0$, since that causes the antisymmetric portion of identity (1) to produce the irregular equation

$$S_{[\mu\nu\lambda;\varepsilon]} \equiv 3/2\Lambda_{a[\mu\nu}\Lambda^a{}_{\lambda\varepsilon]}\ (=0). \tag{15}$$

The equation $\Phi_\mu = 0$ or $\Phi_a = C_a = \text{const}$ cannot be added to Eq. (2), since the irregular equation ($\Phi_\mu = 0$ only for trivial solutions)

$$E_{aa} = \Lambda_{acd}\Lambda_{cda} + [3\sigma + 2\sigma^2(1-n)]\,C_a C_a = 0.$$

appears.

If we now consider the disturbance $\delta h_{a\mu}$: $h_{a\mu} \to h_{a\mu} + \delta h_{a\mu}$, then from Eq. (14) we obtain an expression for $\delta f_{\mu\nu}$ ($\delta f^{\mu\nu} = g^{\mu\varepsilon}g^{\nu\tau}\delta f_{\varepsilon\tau}$, since for the background $f = 0$)

$$(\delta f^{\mu\nu})_{;\nu} = (1-3\sigma)\,\Phi_\nu\,\delta f^{\mu\nu}\ \ ((\delta f_{[\mu\nu})_{;\lambda]} \equiv 0),$$

i.e., for $\sigma = 1/3$ a complete illusion is possible, as though an "independent" field $\delta f_{\mu\nu}$ were immersed in a Riemann space and moved in the standard manner, along a geodesic (in fact, all united by the field $h_\mu{}^a$ and somewhat twisted aside from the metric: for example, Φ_μ cannot be expressed in terms of the metric). This is of importance, since it is just this $f_{\mu\nu}$ which transports energy; the equivalence may be related to the universal ("electromagnetic") character of $T_{\mu\nu}$. If $\sigma \neq 1/3$, then the motion $\delta f_{\mu\nu}$ will be determined not by the metric alone, as is also the case for motion of the component $\delta S_{\mu\nu\lambda}$ for any σ [see Eq. (15)].

The form of stars and planets is close to spherical, but the natural rotation of the stars or planets disrupts spherical symmetry and must induce a field $\Delta S_{\mu\nu\lambda}$. Since the phenomenology of nonweightless material (additional measurement~~s~~ ~~are~~ is required here [5, ~~7~~]) and its interaction with $S_{\mu\nu\lambda}$ are unclear, it is difficult to make any estimates. However, it cannot be excluded that $\Delta S_{\mu\nu\lambda}$ may produce a nonzero field $\Delta f_{\mu\nu}$ [see Eq. (14), possibly of either sign!]. This affect probably produces a contribution to the magnetic field of rotating stars and planets [8-10].

It can be expected that in the Shapiro experiment the field $\Delta S_{\mu\nu\lambda}$ is capable of affecting only the polarization of the packet $\delta f_{\mu\nu}$, since $\Delta S_{\mu\nu\lambda}$ cannot appear in the eiconal equation.

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